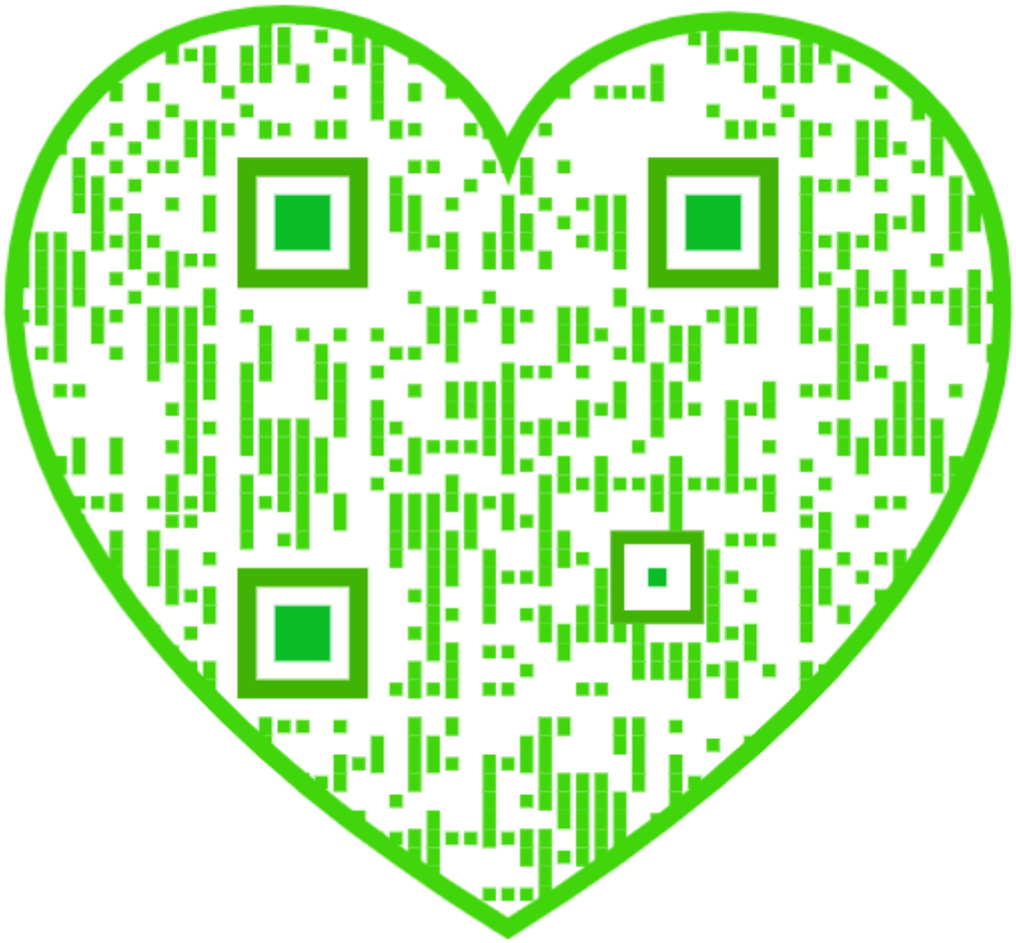

FERRELL'S

ELEMENTARY ARITHMETIC.

BY

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FERRELL'S ARITHMETICS.

Ferrell's Teachers' and Students' Manual of Arithmetic.

165 Pages.

Ferrell's Elementary Arithmetic.

217 + xiii Pages.

Ferrell's Advanced Arithmetic.

408 + xvi Pages.

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PREFACE.

THIS book was prepared primarily as a first book in Arithmetic for those schools in which a text is not put into the hands of the pupil before the beginning of the third school year. It contains the mathematics for the third, fourth, and fifth school years. But, if preceded by Wooster's Arithmetics or their equivalents in the first, second and third years, Parts One and Two may be completed during the fourth year. In that event, only time enough need be taken with Part One to review hastily and to understand the plan of work preparatory to taking up Part Two.

This is the companion book of Ferrell's Advanced Arithmetic, which contains the mathematics for the sixth, seventh and eighth school years.

The author claims for this book the following special features of excellence:

I. THE SPIRAL PLAN OF ARRANGEMENT.—The first part reviews what the pupil has been taught, or has "picked up" before the end of the second school year. It systematizes this knowledge, and adds suitable advanced work. The Second and Third Parts each review the preceding part, and continue to add suitable advanced work.

II. THE UNIFORM AND PSYCHOLOGICAL GRADATION.—The association and sequence of the various subjects have been carefully studied, and the order of the development of the mental

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faculties, as well as the logical order of the science of arithmetic, has been closely followed.

III. The Equation Plan of Solution.—There is set forth in this book a complete and uniform equation plan of solution for all classes of problems coming within its scope.

IV. The Proportion Plan of Solution.—There is presented a new and logical plan by which the proportional equality among the parts of a problem may be easily and accurately stated, and the problem solved.

V. Study of Problems.—Arithmetic is a science of *problems*, as well as a science of *numbers*. Recognizing this fact, the author has made special effort to present a systematic study of the nature and solution of arithmetic problems, and has given about 125 model solutions, fully illustrating the solution of every type of problems coming within the scope of this book. The special study of problems begins on page 33.

The author's greatest hope is, that this book will prove to be a teachable text for the school-room.

J. A. F.

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INTRODUCTION.

TO THE TEACHER:

I. GENERAL DIRECTION. *Do not be afraid you will tell the pupil too much in explaining the facts, principles, definitions, and examples preparatory to his study of processes and problems.* The primary pupil's work in Arithmetic will be sufficiently difficult, even after you have given him all the explanation your time will permit.

II. STUDY OF EXPLANATORY MATTER.—All explanatory matter on any subject, such as signs, definitions, principles, examples, and notes, should be studied and understood by the pupil before he attempts to prepare the exercise following such explanations. This explanatory matter, after the teacher has gone over it with the pupil, emphasizing and adding to the explanations of the book when necessary, should be assigned for study. Too often, definitions and principles are committed to memory without first being understood. The average third or fourth grade pupil will not at first be able unassisted to read understandingly for himself, the notes and explanations in his Arithmetic. Help and teach him to do this, and he will have little trouble in preparing his exercises.

III. BLACK TYPE.—The use of the *black type* is to mark terms, expressions or facts that are of special importance. In the first seventeen articles, the forty-five combinations in black type should be drilled upon, until the pupil can quickly give

the result of every combination. Tabulated work in black type, such as is found on pages 29 and 56, should be thoroughly committed.

IV. THE ABILITY OF THE PUPIL.—The ability of the pupil should be a subject for the teacher's special consideration, both in assigning lessons and in conducting recitations. The articles of this book are not intended to correspond to lessons. No attempt is made by the author to suggest lesson limits. The teacher's judgment, based upon a knowledge of the subject and of the pupil's ability, is the only true guide in the assignment of lessons.

As an example of the way in which the teacher may vary the work to suit the ability of the pupil, take the expression, *3−2+1*, found at the bottom of page 3. In recitation, the teacher may say, "3−2"; pause for the pupil to think, then say, "+1"; or the teacher may call the whole expression, "3−2+1," at once.

V. ORAL AND REVIEW WORK.—Pupils must be taught to think quickly as well as accurately. Both oral work and reviews are valuable in acquiring these results. In oral work, the child's attention is given almost wholly to *thinking*, and is not divided between *thinking* and *other acts*, as it is in written work. For this reason, oral work is excellent for securing speed. Reviews fix the facts, principles and processes in the mind. The mind is thus made familiar with them, and learns to use them with speed and accuracy.

Many of the exercises, intended for written work, may also be used for oral review work.

VI. USE OF DITTO MARKS.—No ditto marks are used in the solutions of this text; yet the author uses them in class work wherever convenient. This matter is discretionary with the teacher. But, if ditto marks are used, the pupil should not be permitted to make a row of miscellaneous dots for ditto marks. Always insist upon neatness as well as system in all solutions.

VII. ANSWERS.—Answers are not put in this book. The author believes that the presence of *answers*, either with the problem or at the end of the book, fosters dependency and lack of self-confidence in the pupil. For teachers, who may need answers to save time and labor in correcting and grading school work, the answers to the *exercises for written work* are printed in a pamphlet, and may be obtained from the publishers.

ELEMENTARY ARITHMETIC.

PART I.

I. SUMS AND DIFFERENCES TO EIGHTEEN.

1. One and Two.

1	one	I
2	two	2
+	plus	+
−	minus	−
=	equals	=

1 hat + 1 hat = 2 hats
2 hats − 1 hat = 1 hat

NOTE.—Teachers should explain the meaning of these equations by the use of the hats themselves. (See Introduction.)

EXERCISE I.

Read and show what these mean:

1 pin + 1 pin = 2 pins. 1 book + 1 book = 2 books.
2 pins − 1 pin = 1 pin. 2 books − 1 book = 1 book.

We have said something about *pins* and *books*. *You may say the same about—*

cats, tops, trees, sticks, dolls, caps.

Is it always true that $1+1=2$?

Copy and fill blanks:

1+1=( )?*	( )+1=2?	2−( )=1?
1+( )=2?	2−1=( )?	( )−1=1?

Write the questions and answers:

Jane had 1 doll, and Kate gave her 1 more. How many dolls did she then have?

Question: 1 doll + 1 doll = ( )?
Answer: 1 doll + 1 doll = 2 dolls.

NOTE.—Teacher should emphasize the fact that the sum is *2 dolls*. Never permit the pupil to omit denominations in concrete work.

Jane had 2 dolls and lost one of them. How many dolls did she then have?

Question: 2 dolls − 1 doll = ( )?
Answer: 2 dolls − 1 doll = 1 doll.

What number added to 1 makes 2?

Question: 1 + ( ) = 2?
Answer: 1 + 1 = 2.

John had 2 cats, but 1 of them died. How many cats had he left?

Ned has 1 ball in each hand. How many balls has he in both?

What number taken from 2 leaves 1?

John had 1 top and found 1 more. How many tops did he then have?

Ned had 2 books, but gave one of them to Nell. How many books had he left?

---

*Read, "One plus one equals *how many?*" or, "One plus one equals *what?*" using *how many* or *what* in place of missing term.

How many added to 1 will make 2?

Make a problem about "1 cat+1 cat=( )?"

Problem: John had 1 cat and Jane had 1 cat. How many cats did they both have?

Make problems about—

1 top+( )=2 tops?
2 dolls−1 doll=( )?
2 dolls−( )=1 doll?

2. Three.

3 three 3

2 hats+1 hat=3 hats
3 hats−2 hats=1 hat

EXERCISE II.

Read and show what these mean:

1 book+2 books=3 books. 3 books−1 book=2 books.

You may say the same about—

cakes, cows, roses, dogs.

Is it always true that **2+1=3?**

*Call results quickly:**

1+2	1+1+1
3−2	2−1+1
2+1	3−2+1
2−1	1+2−1
3−1	2+1−2

*This is an oral exercise. In recitation the teacher should call, "One plus two?" "Three minus two?" and so on, requiring the pupil to respond with the result. Drill on such exercises should be continued until the response from the pupil is quick and accurate.

Write the questions and answers:

John had 3 tops, but he lost 1 of them. How many tops did he then have?

Question: 3 tops − 1 top = ( )?
Answer: 3 tops − 1 top = 2 tops.

Nell had 3 eggs, but she dropped 1 of them and broke it. How many eggs were left?

Cora had 2 apples, and Jane gave her 1 more. How many apples did she then have?

Ruth had 3 cups, but Ned broke 2 of them. How many cups had she left?

NOTE.—The problems should also be used for *oral work.* (See Introduction.)

Make problems about—

3 rats − 2 rats = ( )?
3 nuts − 1 nut = ( )?
2 cows + 1 cow = ( )?

3. Four.

4 four 4

2 + 2 = 4
3 + 1 = 4

NOTE.—Teacher, read Introduction.

EXERCISE III.

Read and show what these mean:

1 book + 3 books = 4 books.
4 books − 2 books = 2 books.

You may say the same about—

pens, birds, chairs, apples.

Call results quickly:

3+1	3−2	4−3+2	1+1+1+1
2+2	4−2	3−2+3	3−2+3−1
1+3	4−1	4−2+1	1+3−2+1
2+1	4−3	2+2−3	2+2−3+1
1+1	3−1	3−1+2	1+2+1−3

Write the questions and answers:

John and James each shot 2 birds. How many birds did both shoot?

Kate has 4 pens in a box. If she should take out 3 of them, how many pens would she have left?

If I have 3 books and buy 1 more, how many books will I then have?

Jane had 4 grains of corn, but the hen ate 2 of them. How many did she then have?

Make problems about —

4 grains−( )=3 grains?
2 pigs+2 pigs=( )?
3 men+1 man=( )?

4. Five.

5 five 5

3+2=5
4+1=5

EXERCISE IV.

Call results quickly:

5−3	2+3	4+1−3	4+1−3+2
5−4	4+1	4−3+2	5−1−3+2
5−2	3+1	2+2+1	3+1−2+3
5−1	2+2	2+3−1	2+2−1+2

Write the questions and answers:

There were 5 slates on a desk, but Kate took 2 of them away. How many slates were left?

I had 1 apple, and Fred gave me 4 more. How many apples did I then have?

There were 5 eggs in a nest, but Nell took out 3 of them. How many eggs were left?

Mary had 2 apples, and Ned gave her 3 more. How many apples had she then?

What must be added to 2 to make 5?

What must be taken from 5 to leave 1?

Make problems about —

5 frogs − 3 frogs = ( )?
3 trees + 2 trees = ( )?
5 marks − ( ) = 1 mark?

5. Six.

6 six 𝔟

3 + 3 = 6
4 + 2 = 6
5 + 1 = 6

EXERCISE V.

Call results quickly:

5+1	6−3	5−4	2+2+2	2+2+1−4
2+4	6−1	4−2	4−2+3	5−3+4−2
3+3	6−4	4−1	6−5+3	6−4+2−1
2+3	6−2	2+2	2+4−2	6−1−3+2
3+1	5−3	4+1	5−2+3	4−2+3−4

Write questions and answers:

James has 2 peaches, and Nell has 4. How many peaches have both?

Ned had 6 apples, but he ate 3 of them. How many apples had he left?

John had 6 toothpicks, but he broke 4 of them. How many toothpicks were left?

John had 6 marbles, but lost 5 of them. How many marbles had he left?

Kate found 5 eggs in one nest and 1 in another. How many eggs did she find?

Nell has 3 beans in each hand. How many beans has she in both hands?

Make problems about —

6 picks − 3 picks = ( )?
5 apples − ( ) = 2 apples?
6 rats − 4 rats = ( )?

6. Seven.

7 seven 7

4 + 3 = 7
5 + 2 = 7
6 + 1 = 7

EXERCISE VI.

Call results quickly:

6+1	2+2	6−2	4−2	2+2+2	7−2+1−4
4+3	1+4	7−3	5−1	3+2+2	5+1−4+2
3+3	2+4	7−5	7−4	5+1+1	6−4+3−1
5+2	5+1	5−3	6−1	3+3+1	7−5+3−4
3+1	1+3	7−1	5−2	4+1+2	7−6+4−3
3+2	2+5	6−3	6−4	7−5+3	4+3−6+2

Write questions and answers:

Dan had 7 hens, but a fox caught 1 of them. How many hens had he left?

How many nails must be added to 3 nails to make 7 nails?

Seven apples are how many more than 5 apples?

James had 5 nuts, but he ate 1 and lost 3. How many nuts did he then have?

Question: 5 nuts − 1 nut − 3 nuts = ( )?
Answer: 5 nuts − 1 nut − 3 nuts = 1 nut.

Nell had 2 roses, and Jane gave her 5 more. How many roses had she then?

Make problems about:

5 hats+1 hat−4 hats=( )?
7 pins−6 pins+4 pins=( )?
4 apples+3 apples−5 apples=( )?
2 nuts+4 nuts−3 nuts=( )?

7. Eight.

8 eight 8

4+4=8 **6+2=8**
5+3=8 **7+1=8**

EXERCISE VII.

Copy, and fill blanks:

4 pins+4 pins=( )?
6 pins+( )=8 pins?
( )+5 pins=8 pins?
1 pin+7 pins=( )?
2 pins+( )=8 pins?
( )+6 pins=8 pins?
3 pins+( )=8 pins?

8 pins−6 pins=( )?
( )−5 pins=3 pins?
8 pins−1 pin=( )?
8 pins−( )=4 pins?
8 pins−7 pins=( )?
8 pins−( )=6 pins?
( )−3 pins=5 pins?

NOTE.—In copying, write the terms complete.

Write questions and answers:

John had 4 apples; Charles gave him 4 more. How many apples did he then have?

Nell has 3 roses, and Jane has 5 roses. How many roses have both?

John, James and Fred have 8 rabbits. If John has 2 rabbits and James has 3 rabbits, how many has Fred?

I have 8 dollars. If I spend 5 dollars for a coat, how many dollars will I have left?

Call results quickly:

4+4	3+2	8−3	8−1	8−5+2	7−5+4−3
5+2	4+3	7−5	4−2	8−3+1	4+3−2+1
3+5	1+6	8−4	6−3	6−3+4	3+3−5+6
2+4	2+6	6−2	5−1	5−4+6	8−6+2−3
3+3	1+7	7−4	7−3	3−2+5	5−3+1−3
6+2	5+3	8−5	8−2	4+3−5	2+4−3+5
7+1	2+4	6−3	5−3	7−4+5	8−5−2+3

8. Nine.

9 nine 9

5 + 4 = 9 **7 + 2 = 9**
6 + 3 = 9 **8 + 1 = 9**

EXERCISE VIII.

Call results quickly:

4+5	6+3	9−5	7−2	3+3+3	7+2−6
5+4	2+5	7−5	9−4	4+5−6	5+2−4
7+2	1+8	6−5	7−3	3+6−2	3+4−5
5+3	3+5	8−5	9−2	9−7+4	2+4−3
7+1	4+4	9−3	8−4	9−5+4	9−8+4
5+2	3+3	6−3	9−1	9−7+2	7−2+4

7−1+3	3+6−2	9−8+6−3+5	3+4−2+4−5
3+5−2	2+2+5	7+1−4+2−3	5+4−8+6−7
2+5+2	4+4+1	2+4−3+6−4	9−5+3−7+9

Write questions and answers:

Ray had 9 grains of corn, but fed 4 to the hen. How many grains were left?

Nell had 6 pencils in one hand and 3 in the other. How many pencils had she in both?

How many added to 7 make 9?

How many added to 1 make 9?

James had 9 sticks, but he burned 5 of them. How many sticks were left?

After 6 boys went out of the room, there were 3 left. How many boys were there at first?

9. Ten.

0 zero, naught, (nothing) 0

10 ten

The figure on the left shows **1 ten,** and the figure on the right shows **nothing (0)** more.

5 + 5 = 10 **7 + 3 = 10**
6 + 4 = 10 **8 + 2 = 10**
9 + 1 = 10

EXERCISE IX.

Copy and fill blanks:

5 grains + 5 grains = ( )?
6 grains + ( ) = 10 grains?
( ) + 3 grains = 10 grains?
8 grains + 2 grains = ( )?
( ) + 1 grain = 10 grains?
10 grains − 1 grain = ( )?
10 grains − ( ) = 8 grains?

( ) − 3 grains = 7 grains?
10 grains − ( ) = 5 grains?
10 grains − 4 grains = ( )?
10 grains − 6 grains = ( )?
( ) − 7 grains = 3 grains?
10 grains − 8 grains = ( )?
10 grains − ( ) = 1 grain?

10 grains − 7 grains + 3 grains − 5 grains = ( )?
5 grains + 5 grains − 6 grains − 2 grains = ( )?
7 grains − 5 grains + 8 grains − 10 grains = ( )?

Write questions and answers:

One hen ate 6 grains of corn, and another ate 4 grains. How many grains did both eat?

James caught 10 birds in a trap, but 3 of them got away. How many birds were left?

There are 5 books on the table, and 5 on the chair. How many books in all?

After Ned killed 2 birds of a flock, 8 flew away. How many birds were in the flock at first?

Nell had 10 cents, but she bought a slate for 9 cents. How many cents had she left?

Two men dig a well for $10. If one man gets $7, how much does the other get?

NOTE.—$10, ten dollars; $7, seven dollars. The sign, $, is used for *dollar* or *dollars*.

10. Eleven.

11

These figures show **1 ten** and **1, eleven.**

6 + 5 = 11 **8 + 3 = 11**
7 + 4 = 11 **9 + 2 = 11**
10 + 1 = 11

EXERCISE X.

Call results quickly:

6+5	3+8	11−9	11−3
4+5	7+4	11−6	10−5
6+4	8+2	11−5	7−6
5+6	9+2	11−8	10−3
5+5	10+1	11−4	9−4

3+7−6	11−9+6−3	8+2− 6+3− 5
4+7−4	11−7+5−8	11−7+ 4−8+11
5+4−6	9−6+8−3	9−5+ 6−3+ 2
11−9+5	4+3+4−9	10−8+ 3−4+ 7
11−7+6	5+5−7+8	7+4−10+9− 8
9−6+8	7−4+6+2	8−5+ 6−7+ 4

Write questions and answers:

There are 5 pigs in one pen, and 6 in another. How many pigs in both?

Ned had 11 pencils, and sold three of them. How many had he left?

Papa gave me 7 books, and sister gave me 4. How many books did I then have?

I bought a slate for 9 cents, and had 2 cents left. How many cents had I at first?

There are four bundles of sticks: in the first there are 4; in the second, 3; in the third, 2; in the fourth, 1. How many sticks in all?

11. Twelve.

12

These figures show **1 ten** and **2, twelve.**

6 + 6 = 12 **8 + 4 = 12**
7 + 5 = 12 **9 + 3 = 12**

EXERCISE XI.

Call results quickly:

7+5	2+2	9− 4	12−10	12−9+ 6
6+6	1+1	12−11	12− 5	9+2− 7
4+4	3+5	12− 9	12− 3	7+3− 5
5+5	7+4	12− 6	12− 1	6+6− 9
3+3	10+2	12− 8	12− 2	8+4−11

11− 9+10	5+ 5+ 2	12− 9+ 8−3+4
12− 7+ 5	4+ 4+ 4	10− 9+11−7+3
3+ 9− 7	3+ 3+ 6	12−10+ 8−9+6
1+10− 8	12−10+ 6	9+ 3−11+6−5
6+ 6−11	11− 9+10	12− 9+ 4−2+7

Write questions and answers:

John is 7 years old; James is 12. What is the difference between their ages?

There were 12 little girls in a class, but 6 of them were absent one day. How many girls were at school?

Kate had 12 eggs, but she let some of them fall and break. If she had 8 eggs left, how many were broken?

A man has 12 pigs in two pens. If there are 3 pigs in one pen, how many are in the other?

Make problems about —

12 men−6 men=( )?
12 trees−4 trees=( )?
7 apples+5 apples=( )?
9 books+( )=12 books?

12. Thirteen.

13

These figures show **1 ten** and **3, thirteen.**

7 + 6 = 13
8 + 5 = 13
9 + 4 = 13

EXERCISE XII.

Call results quickly:

7+6	10+3	11− 9	12−6	10−7+9
5+7	9+4	13−10	10−5	13−8+4
6+5	7+2	12− 8	13−4	5+7−4
9+4	8+5	9− 6	13−2	6+7−5
7+3	6+7	10− 7	13−3	4+9−7

8+ 4−6	12−6+7	13− 8+ 3−5+7
9− 4+8	11+1−9	13− 2+ 1−9+4
10− 5+4	13−5+2	13−11+ 6−5+3
13−11+6	10−6+7	12− 6+ 4−8+2
13− 8+3	11−4+6	11−10+12−7+4

Add by 2's from 1 to 13.

Call results: 3, 5, 7, 9, 11, 13.

Subtract by 2's from 13 to 1.

Call results: 11, 9, 7, 5, 3, 1.

Add by 3's from 0 to 12. Subtract by 3's from 12 to 0.

Add by 4's from 1 to 13. Subtract by 3's from 13 to 1.

Write questions and answers:

There are 5 pigs in one pen and 8 pigs in another. How many in both?

Kate had 13 apples, but she gave 6 of them to James and 4 to Nell. How many had she left?

I paid 7 dollars for clothes and 6 dollars for books. How many dollars did I spend?

How many must be added to 4 to make 13?

Make problems about—

10 men−7 men+( )=8 men?
8 books+5 books−10 books=( )?
$7−$4+$10=( )?
$9+$2− $4=( )?
6 pens+4 pens−7 pens=( )?

13. Fourteen.

14

These figures show **1 ten and 4, fourteen.**

7 + 7 = 14
8 + 6 = 14
9 + 5 = 14

EXERCISE XIII.

Write questions and answers:

James has 7 marbles in each of his two pockets. How many has he in both pockets?

Ned killed 6 quails out of a flock of 14. How many quails got away?

A man spent $10 for clothes and $4 for books. How much did he spend for both?

Jane had 14 roses, and gave away all but 5. How many roses did she give away?

Call results quickly:

Add by 2's from 0 to 14.	Add by 7's from 0 to 14.
Add by 3's from 2 to 14.	Subtract by 2's from 14 to 0.
Add by 4's from 2 to 14.	Subtract by 3's from 14 to 2.
Add by 5's from 4 to 14.	Subtract by 4's from 14 to 2.
Add by 6's from 2 to 14.	Subtract by 7's from 14 to 0.

14. Fifteen.

15

These figures show **1 ten** and **5, fifteen.**

$$8+7=15$$
$$9+6=15$$

EXERCISE XIV.

Call results quickly:

15−10	12+3	15−2	15− 9
7+ 8	5+9	15−4	15− 5
5+ 8	8+6	15−6	15− 8
7+ 6	11+3	15−7	15− 1
4+11	13+2	15−8	15−11

15−6+ 3	13−7+4	15−10+6
12+3− 9	14−9+5	13−12+3
11+2− 8	5+8−3	14− 9+7
9+5−10	7+7−2	8+ 7−2
12−7+ 5	5+5+5	9+ 5−4

Add by 5's from 0 to 15. How many 5's in 15?
Add by 3's from 0 to 15. How many 3's in 15?
Subtract by 3's from 15 to 0. Subtract by 4's from 15 to 3.
Subtract by 2's from 15 to 1. Subtract by 5's from 15 to 0.

Write questions and answers:

Mary picked 4 quarts of berries, John picked 7 quarts, and May picked 3 quarts. How many quarts did all pick?

John picked 9 quarts of berries, and May picked 6 quarts, but John spilled 4 quarts of his and they were spoiled. How many quarts did both then have?

How many apples must be added to 10 apples to make 15 apples?

James had 15 apples and ate 6; Jane had 12 apples, and ate of them till she had left as many as James. How many did she eat?

NOTE.—First, find how many apples James had left; secondly, find how many Jane must eat.

After adding 7 tops to 8 tops, how many must be taken away to leave 6 tops?

15. Sixteen.

16

These figures show **1 ten** and **6, sixteen.**

8 + 8 = 16
9 + 7 = 16

EXERCISE XV.

Call results quickly:

8+8	9+7	16− 5	15−10
6+6	12+4	16− 8	15− 8
7+7	13+2	16−12	15− 4
5+5	10+6	16−10	15−12
4+4	7+9	16−13	15−11
3+3	7+5	16− 6	13− 7

Add by 2's from 0 to 16. How many 2's in 16?
Add by 4's from 0 to 16. How many 4's in 16?
Add by 8's from 0 to 16. How many 8's in 16?

16−12+11− 9+8−13+12− 5+7=( )?
5+ 8− 9+12−7+ 3− 8+10+2=( )?
3+ 5+ 7− 6−5+12− 9+ 7−2=( )?
5+ 9+ 2− 8−5+ 3+ 8− 7+5=( )?

Write questions and answers:

In a class of 16 pupils there are 8 girls. How many boys are there?

A man had 7 sheep and bought 9 more. How many did he then have?

Sixteen men engage to work for a man, but 6 of them get sick. How many are left to work?

A lady sold 6 chickens to one man, 8 to another, and had 2 left. How many had she at first?

16. Seventeen.

17

These figures show **1 ten** and **7, seventeen.**

$$9+8=17$$

EXERCISE XVI.

Call results quickly:

9+8	7+10	17− 9	16−11
9+6	12+ 5	17−12	17− 4
9+5	11+ 4	17− 6	15− 2
9+7	10+ 6	17−10	13− 9
9+4	9+ 7	17− 8	12− 7

17−9− 5+12−9+6−10
11+5−14+ 6+5−7+ 6
5+8− 7+ 6+4−3+ 4
2+4+ 9− 3−3−3− 3
7+7+ 3−10+7−6− 8

Write questions and answers:

From what number must 9 be subtracted to leave 8?

How many added to 7 make 17?

A man bought a coat for $8, a vest for $3, and a pair of pants for $6. How much did they all cost him?

If 9 be taken from a certain number, the remainder is 8. What is the number?

Three trains passed by. The first had 4 cars, the second had 7 cars, and the third had 6 cars. How many cars in all?

George is 17 years old. How old was he 9 years ago?

I had 10 cents, and my mamma gave me 6 cents more. I then lost some, and found that I had 5 cents left. How many did I lose?

I had $17. I bought a watch for $11, and a hat. How much did the hat cost if I had $4 left?

17. Eighteen.

18

These figures show **1 ten** and **8, eighteen.**

9 + 9 = 18

EXERCISE XVII.

Call results quickly:

9+ 9	8+9	18− 9	18− 2	3+3+ 3+3+ 3+ 3
7+11	10+8	18−15	18−17	12+6−14+8− 7+ 5
9+ 6	7+7	18−14	18−13	18−6− 6−6+ 9+ 9
11+ 5	13+5	17− 6	17− 9	17+1− 9+5− 7− 7
12+ 6	11+6	15−11	18−10	7+8− 6−7+11−13

How many 2's in 4? in 6? in 8? in 10? in 12? in 14? in 16? in 18?

How many 3's in 6? in 9? in 12? in 15? in 18?

How many 4's in 8? in 12? in 16?

How many 5's in 10? in 15?

How many 6's in 12? in 18?

How many 7's in 14? How many 8's in 16? How many 9's in 18?

Write questions and answers:

What number subtracted from 18 will leave 7?

What number added to nine will make 18?

Charles has 17 apples: he gives Mary 5. How many apples can he give to George and still have 6 left?

How much larger is 18 than 14?

James is 18 years old; John is 3 years younger than James, and Nell is 4 years younger than John. How old is Nell?

There are 18 birds in a flock. Charles kills 5 and George kills 7 of them. How many birds are left?

James spends $8 for books and $10 for clothes. How much does he spend in all?

Make problems about:

7 books+11 books=( )?
18 books−( )=9 books?
18 books−15 books=( )?
8 books+( )=18 books?

II. NOTATION AND NUMERATION TO NINETY-NINE.

18. Units and Tens.—A **unit** is a single thing, or one. In reading and writing numbers, the first place, or the one on the right, is for units, and is called **Units' Place.** Numbers up to nine are written in units' place only. Thus:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
zero,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

In numbers of more than nine units, the units are grouped as far as possible into groups of ten each. Groups of tens are named as follows:

2 tens, **twenty,**	6 tens, **sixty,**
3 tens, **thirty,**	7 tens, **seventy,**
4 tens, **forty,**	8 tens, **eighty,**
5 tens, **fifty,**	9 tens, **ninety.**

The place next to the left of units is for *tens*, and is called **Tens' Place.**

In writing numbers, put the number of tens in tens' place and the number of units in units' place. If there are no units, put "0" in units' place.

EXERCISE XVIII.

Write each number with figures:

Ten, 1 ten and 0 units.
Written: 10.

Eleven, 1 ten and 1 unit.
Written: 11

Twelve, 1 ten and 2 units.
Thirteen, 1 ten and 3 units.
Fourteen, 1 ten and 4 units.
Fifteen, 1 ten and 5 units.
Sixteen, 1 ten and 6 units.
Seventeen, 1 ten and 7 units.
Eighteen, 1 ten and 8 units.
Nineteen, 1 ten and 9 units.

Read each number:

20, 2 tens and 0 unit.
Read: Twenty.

21, 2 tens and 1 unit.
Read: Twenty-one.

22, 2 tens and 2 units.
23, 2 tens and 3 units.
24, 2 tens and 4 units.
25, 2 tens and 5 units.
26, 2 tens and 6 units.
27, 2 tens and 7 units.
28, 2 tens and 8 units.
29, 2 tens and 9 units.

Write and read the numbers—

From **Thirty** to **Thirty-nine.**
Written: 30, 31, 32, 33, *etc.*

From **Forty** to **Forty-nine.**
From **Fifty** to **Fifty-nine.**
From **Sixty** to **Sixty-nine.**
From **Seventy** to **Seventy-nine.**
From **Eighty** to **Eighty-nine.**
From **Ninety** to **Ninety-nine.**

Read rapidly both ways:

13	15	18	14	19	16	17	11	12	10
26	23	28	21	29	20	22	27	25	24
30	38	36	31	39	35	37	32	34	33
42	45	48	43	47	49	46	44	40	41
52	54	59	51	53	55	57	50	58	56
66	60	67	61	63	65	62	64	69	68
79	78	73	71	76	72	77	74	70	75
82	88	83	80	89	87	85	81	86	84
92	99	98	96	97	93	95	94	90	91
17	25	94	76	50	48	35	88	61	90

Write:

Twenty-one	Fifty-four	Thirty-nine	Thirty-seven
Eleven	Forty-two	Eighty-one	Fifty-nine
Seventeen	Twenty-seven	Thirteen	Forty-one
Forty-four	Nineteen	Forty-three	Sixty-nine
Seventy	Forty-nine	Thirty-three	Sixty
Eighty-nine	Thirty	Twenty-two	Twenty-five
Forty-six	Forty-seven	Fifty-five	Sixty-two
Twenty-five	Ninety-four	Seventy-seven	Thirty-one
Ninety-seven	Seventy-two	Ninety-eight	Ninety-three
Sixty-three	Ninety	Eighty-two	Forty-five
Thirty-eight	Sixty-six	Seventy-three	Eighty-four

III. SUMS AND DIFFERENCES TO NINETY-NINE.

19. Twos, Threes, and Fours.

EXERCISE XIX.

Call results rapidly:

1. Add by 2's from 1 to 99.
2. Add by 2's from 0 to 98.
3. Add by 3's from 0 to 99.
4. Add by 3's from 1 to 97.
5. Add by 3's from 2 to 98.
6. Add by 4's from 0 to 96.
7. Add by 4's from 1 to 97.
8. Add by 4's from 2 to 98.
9. Add by 4's from 3 to 99.
10. Subtract by 2's from 99 to 1.
11. Subtract by 2's from 98 to 0.
12. Subtract by 3's from 99 to 0.
13. Subtract by 3's from 97 to 1.
14. Subtract by 3's from 98 to 2.
15. Subtract by 4's from 96 to 0.
16. Subtract by 4's from 97 to 1.
17. Subtract by 4's from 98 to 2.
18. Subtract by 4's from 99 to 3.

Copy and add; also add orally from book:

19.	*20.*	*21.*	*22.*	*23.*	*24.*	*25.*	*26.*	*27.*	*28.*
4	3	2	1	4	4	2	4	4	2
3	2	3	2	4	3	4	3	3	4
2	3	4	4	2	0	3	3	4	1
20	21	22	23	24	35	36	37	38	39

NOTE.—Add upward. Ex. 19, say: "20, 22, 25, 29," calling results only, after the first number. "*Carrying*" need not be considered here.

29.	*30.*	*31.*	*32.*	*33.*	*34.*	*35.*	*36.*	*37.*	*38.*
4	2	2	4	3	4	3	3	1	4
2	4	4	4	1	3	2	3	3	3
2	2	0	3	4	4	0	4	4	2
45	44	43	42	41	59	58	57	56	50

39.	*40.*	*41.*	*42.*	*43.*	*44.*	*45.*	*46.*	*47.*	*48.*
4	4	3	3	2	2	3	4	4	1
3	4	2	2	0	2	4	3	4	3
2	3	3	1	4	3	3	2	2	4
60	61	62	63	64	75	76	78	79	77

49.	*50.*	*51.*	*52.*	*53.*	*54.*	*55.*	*56.*	*57.*
80	91	82	93	84	95	86	92	88
1	1	3	4	0	0	3	2	4
2	4	2	0	4	0	2	2	1
3	1	4	2	4	3	2	2	3

NOTE.—Add downward. Ex. 49, say: "80, 81, 83, 86."

Copy and find results; also use as an oral exercise:

58.	*59.*	*60.*	*61.*	*62.*	*63.*	*64.*	*65.*	*66.*
33	44	55	56	67	78	89	91	13
−2	+4	−2	−4	+2	+1	−4	−2	−1
+4	−3	+4	+2	−4	−4	+1	+3	+4

NOTE.—Call only results, after the first number. Ex. 58, say: "33, 31, 35."

67.	*68.*	*69.*	*70.*	*71.*	*72.*	*73.*	*74.*	*75.*
95	88	77	66	55	44	33	22	17
−4	+3	+1	−4	−2	−3	−2	+1	+4
+2	−4	−3	+3	+4	+1	+1	−4	−2

76.	*77.*	*78.*	*79.*	*80.*	*81.*	*82.*	*83.*	*84.*
47	59	62	84	97	81	95	76	49
−1	+2	+1	+4	+1	−4	−1	−3	−1
+3	−4	−3	−2	−3	+2	+4	+1	+3

85. I paid 53¢ for a book, 4¢ for a pencil, and 3¢ for pens. How much did all cost?

3¢
4¢
53¢
60¢, answer.

NOTE.—The sign ¢ is used for *cent* or *cents*.

86. After giving 4 oranges to Nell, 3 to Charles, and 2 to May, Henry has 39 oranges left. How many had he at first?

87. A baker had 33 loaves of bread left after selling 2 loaves to Mrs. Jones and 4 to Mrs. Brown. How many loaves had he at first?

88. On Monday, I walked 3 miles; on Tuesday, I walked 4 miles; on Wednesday, I walked 17 miles. How far did I walk in the three days?

89. A man bought a horse for $78, a sheep for $4, and a pig for $2. How much did he pay for all?

90. I bought a suit of clothes for $19, a hat for $2, and a pair of shoes for $3. How much did they all cost?

91. A stove cost $24, and a ton of coal cost $3. After buying these I had $4 left. How much money had I at first?

92. Mr. Jones had 70 sheep. After 2 of them died, he bought 4 more. How many sheep did he then have?

20. Fives, Sixes, and Sevens.

EXERCISE XX.

Call results rapidly:

1. Add by 5's from 0 to 95.
2. Add by 5's from 1 to 96.
3. Add by 5's from 2 to 97.
4. Add by 5's from 3 to 98.
5. Add by 5's from 4 to 99.
6. Subtract by 5's from 99 to 4.
7. Subtract by 5's from 98 to 3.
8. Subtract by 5's from 95 to 0.
9. Subtract by 5's from 97 to 2.
10. Subtract by 5's from 96 to 1.

11. Add by 6's from 0 to 96.
12. Add by 6's from 1 to 97.
13. Add by 6's from 2 to 98.
14. Add by 6's from 3 to 99.
15. Add by 6's from 4 to 94.
16. Add by 6's from 5 to 95.
17. Add by 7's from 0 to 98.
18. Add by 7's from 1 to 99.
19. Add by 7's from 2 to 93.
20. Add by 7's from 3 to 94.
21. Add by 7's from 4 to 95.
22. Add by 7's from 5 to 96.
23. Add by 7's from 6 to 97.

24. Subtract by 6's from 95 to 5.
25. Subtract by 6's from 94 to 4.
26. Subtract by 6's from 99 to 3.
27. Subtract by 6's from 96 to 0.
28. Subtract by 6's from 98 to 2.
29. Subtract by 6's from 97 to 1.
30. Subtract by 7's from 97 to 6.
31. Subtract by 7's from 96 to 5.
32. Subtract by 7's from 95 to 4.
33. Subtract by 7's from 94 to 3.
34. Subtract by 7's from 93 to 2.
35. Subtract by 7's from 99 to 1.
36. Subtract by 7's from 98 to 0.

Copy and find results; also use as an oral exercise:

37.	*38.*	*39.*	*40.*	*41.*	*42.*	*43.*	*44.*	*45.*
26	27	30	35	37	29	38	44	43
−7	+7	−5	−7	+6	−6	+7	+4	+7
+6	−5	+7	+5	−5	−5	−6	+5	−6
−5	+6	+6	−6	−7	−7	−5	+6	+3

46.	*47.*	*48.*	*49.*	*50.*	*51.*	*52.*	*53.*	*54.*
47	42	40	55	58	56	64	67	69
+6	−7	+7	+5	−4	+7	−4	+7	−6
+5	−6	+5	−7	−6	+6	−4	+7	−7
−7	−5	+6	+6	+7	+3	−4	+7	+6

55.	*56.*	*57.*	*58.*	*59.*	*60.*	*61.*	*62.*	*63.*
77	74	78	80	87	95	93	89	99
+7	+3	+7	+5	+7	−7	+6	−7	−7
+6	+6	+5	+3	−3	−6	−5	+6	−6
+5	+5	−6	−7	+6	−5	−7	−5	+4

64.	*65.*	*66.*	*67.*	*68.*	*69.*	*70.*	*71.*	*72.*
89	83	47	58	65	72	91	82	66
+4	−6	+7	+7	−5	−7	+6	+4	+5
+6	−4	+6	−6	−4	+3	−4	−7	+2
−5	+7	+5	−5	+7	−6	−7	+5	−6

73.	*74.*	*75.*	*76.*	*77.*	*78.*	*79.*	*80.*	*81.*
49	40	60	71	79	77	95	82	99
+7	+5	−3	+7	−7	−4	−5	+7	−6
+7	−2	+6	−6	−6	+7	−3	−5	−4
+7	+7	+7	−3	−5	−3	+7	−3	+7

82. Charles paid $35 for a watch. At one time he paid $5 for repairs, at another $4. How much did the watch cost altogether?

83. James bought a typewriter for $75, a set of books for $6, and a table for $4. What did all cost him?

84. Nell's dress cost $12, her cloak $7, her hat $5. How much did all cost?

85. I now have $83. I bought a coat for $8, a vest for $4, and a pair of trousers for $5. How much money did I have at first?

86. George had 48¢, and found 6¢. After spending 7¢ for a ball and 5¢ for apples, how much had he left?

87. A man had 67 bushels of apples. He sold 6 bushels to one man, 7 bushels to another. He then bought 5 bushels. How many bushels did he then have?

88. Thirty-eight men are at work on a bridge; 6 more are employed the same day, and 7 more the next day; but 5 of the old hands get sick. How many are then left to work?

89. I had $73, and found $5 more. I then spent $4 for clothing and $7 for rent. How much money had I left?

90. I have 57 relatives in Missouri, 7 in California, 6 in Kansas, 7 in Illinois. How many have I in all?

21. Eights and Nines.

EXERCISE XXI.

Call results rapidly:

1. Add by 8's from 0 to 96.
2. Add by 8's from 1 to 97.
3. Add by 8's from 2 to 98.
4. Add by 8's from 3 to 99.
5. Add by 8's from 4 to 92.
6. Add by 8's from 5 to 93.
7. Add by 8's from 6 to 94.
8. Add by 8's from 7 to 95.
9. Add by 9's from 0 to 99.
10. Add by 9's from 1 to 91.
11. Add by 9's from 2 to 92.
12. Add by 9's from 3 to 93.
13. Add by 9's from 4 to 94.
14. Add by 9's from 5 to 95.
15. Add by 9's from 6 to 96.
16. Add by 9's from 7 to 97.
17. Add by 9's from 8 to 98.
18. Subtract by 8's from 97 to 1.
19. Subtract by 8's from 96 to 0.
20. Subtract by 8's from 98 to 2.
21. Subtract by 8's from 92 to 4.
22. Subtract by 8's from 99 to 3.
23. Subtract by 8's from 94 to 6.
24. Subtract by 8's from 93 to 5.
25. Subtract by 8's from 95 to 7.
26. Subtract by 9's from 92 to 2.
27. Subtract by 9's from 91 to 1.
28. Subtract by 9's from 99 to 0.
29. Subtract by 9's from 98 to 8.
30. Subtract by 9's from 97 to 7.
31. Subtract by 9's from 95 to 5.
32. Subtract by 9's from 94 to 4.
33. Subtract by 9's from 96 to 6.
34. Subtract by 9's from 93 to 3.

Copy and add; also use as an oral exercise:

35.	*36.*	*37.*	*38.*	*39.*	*40.*	*41.*	*42.*	*43.*
25	27	29	36	33	30	41	39	44
+9	−6	−5	+6	−9	+7	−4	+9	+6
−6	+7	+6	+9	+7	+9	−6	−6	−4
−5	−5	+8	−7	+6	−6	+7	−5	−7
+4	+8	−9	−5	+4	−8	+9	+8	−9

44.	*45.*	*46.*	*47.*	*48.*	*49.*	*50.*	*51.*	*52.*
48	45	46	51	53	57	59	60	66
+9	+5	+9	+9	+9	−9	+6	+9	−7
−8	+7	+8	−8	+8	−7	−8	+8	+5
−6	−6	−5	−6	+7	−6	+7	−3	−6
+5	−8	−7	+5	+6	−5	+9	−9	+8

53.	*54.*	*55.*	*56.*	*57.*	*58.*	*59.*	*60.*	*61.*
67	65	64	69	68	71	72	74	78
−6	+7	−9	+7	+8	−9	−3	−9	−9
−7	+5	−8	+3	−4	−8	+9	−7	−9
+9	−9	−7	+0	+9	+5	−5	+8	−9
−8	−6	−5	+9	−6	+7	+8	+8	−8

62.	*63.*	*64.*	*65.*	*66.*	*67.*	*68.*	*69.*	*70.*
73	88	87	86	95	97	93	92	90
+8	−7	+8	−9	−7	−6	−7	−9	+8
+6	−9	−6	−7	+5	+8	+5	−3	−5
+7	+4	+7	+6	−7	−3	−8	+7	−6
+5	−9	−9	−4	−9	−7	+6	+5	+9

71. A man had 57 cattle in one pasture, 8 in another, 6 in another, but 9 of them died. How many cattle had he left?

72. I have 72 books in a case, 9 books on one table, and 5 on another. If I lend 8 of them, how many will I have left?

73. Mr. Brown collects $93 on Monday, he spends $9 on Tuesday, he collects $6 on Wednesday, $5 on Thursday, on Friday he spends $8. How much money has he left?

74. I bought a buggy for $65, a washing machine for $9, a plow for $8, and a table for $6. How much did they all cost?

75. Charles paid his board for 3 months, $48; for a coat, $8; for a pair of trousers, $5; for a vest, $4; for expenses, $6. How much did he pay for all?

76. On a certain farm there are 84 acres in wheat, 8 acres in oats, 9 acres in corn, and 6 acres in alfalfa. How many acres in all?

77. I owed a bill of $98. At one time I paid $7, at another $5, at another $9. How much did I then owe?

78. A man planted an orchard of 76 trees. He afterwards planted 8 trees at one time and 7 at another time. He then found that 9 trees had died. How many had he left?

IV. PRODUCTS AND PARTS TO EIGHTY-ONE.

× times* ×

22. Two Times and Halves.—If we add together two numbers of the same size, the sum will be **Two Times** (2×) one of the numbers. Thus:

3 books+3 books=6 books; then,

2×3 books=6 books

If we divide a number into two equal parts, one of the parts will be **One-half** (½) of the number; two parts, **Two-halves** (2/2), or all of it. Thus:

4 books=2 books+2 books; then,

½ of 4 books=2 books

2/2 of 4 books=4 books

Teacher, go over the following with the pupils in class before giving the exercise:

2×0=0	**2×5=10**	**½ of 0=0**	**½ of 10=5**
2×1=2	**2×6=12**	**½ of 2=1**	**½ of 12=6**
2×2=4	**2×7=14**	**½ of 4=2**	**½ of 14=7**
2×3=6	**2×8=16**	**½ of 6=3**	**½ of 16=8**
2×4=8	**2×9=18**	**½ of 8=4**	**½ of 18=9**

EXERCISE XXII.

Copy and fill blanks:

1.	*2.*	*3.*	*4.*
2×0=( )?	2×2=( )?	½ of 0=( )?	½ of 14=( )?
2×4=( )?	2×5=( )?	½ of 6=( )?	½ of 18=( )?
2×1=( )?	2×9=( )?	½ of 12=( )?	½ of 8=( )?
2×6=( )?	2×7=( )?	½ of 4=( )?	½ of 16=( )?
2×3=( )?	2×8=( )?	½ of 2=( )?	½ of 10=( )?

*"1×" may be read *one time* or *once*; "2×," *two times* or *twice*.

Call results quickly:

5.	6.	7.	8.
2×2	2×9	$\frac{1}{2}$ of 0	$\frac{1}{2}$ of 18
2×0	2×3	$\frac{1}{2}$ of 8	$\frac{1}{2}$ of 14
2×5	2×6	$\frac{1}{2}$ of 12	$\frac{1}{2}$ of 4
2×7	2×4	$\frac{1}{2}$ of 2	$\frac{1}{2}$ of 16
2×8	2×1	$\frac{1}{2}$ of 6	$\frac{1}{2}$ of 10

9. A number has how many halves?

10. Is $2\times\frac{1}{2}$ of a number equal to all of it?

Answer: Yes; $2\times\frac{1}{2}$ of number $=\frac{2}{2}$ of number, or all of it.

Write questions and answers:

11. Jane had 2 birds, and John gave her as many more. How many birds did she then have?

Question: 2×2 birds $=(\ )$?
Answer: 2×2 birds $=4$ birds.

12. Kate has 8 apples, and Ned has $\frac{1}{2}$ as many. How many apples has Ned?

Question: $\frac{1}{2}$ of 8 apples $=(\ )$?
Answer: $\frac{1}{2}$ of 8 apples $=4$ apples.

13. George has 5 books, and James has twice (2 times) as many. How many books has James?

14. Ida had 18 eggs, but she let them fall, and $\frac{1}{2}$ of them were broken. How many were broken?

15. Sarah has 7 roses, and Ida has twice as many. How many roses has Ida?

16. Ned carries 16 sticks of wood, and George carries $\frac{1}{2}$ as many. How many sticks does George carry?

23. Three Times and Thirds.

If we add together three numbers of the same size, the sum will be **Three Times** one of the numbers. Thus:

$$4+4+4=12; \text{ then,}$$
$$\mathbf{3 \times 4 = 12.}$$

If we divide a number into three equal parts, one of the parts will be **One-third** (⅓) of the number; two parts, **Two-thirds** (⅔); and all three parts, **Three-thirds** (⅗). Thus:

$$9=3+3+3; \text{ then,}$$
$$\mathbf{\frac{1}{3} \text{ of } 9 = 3, \quad \frac{2}{3} \text{ of } 9 = 6, \quad \frac{3}{3} \text{ of } 9 = 9.}$$

Remember, that $\frac{2}{3}$ is 2 times $\frac{1}{3}$, and $\frac{3}{3}$ is 3 times $\frac{1}{3}$.

Teacher, go over the following with the pupils in class before giving the exercise:

$3 \times 0 = 0$	$3 \times 5 = 15$	$\frac{1}{3}$ of $0 = 0$	$\frac{1}{3}$ of $15 = 5$
$3 \times 1 = 3$	$3 \times 6 = 18$	$\frac{1}{3}$ of $3 = 1$	$\frac{1}{3}$ of $18 = 6$
$3 \times 2 = 6$	$3 \times 7 = 21$	$\frac{1}{3}$ of $6 = 2$	$\frac{1}{3}$ of $21 = 7$
$3 \times 3 = 9$	$3 \times 8 = 24$	$\frac{1}{3}$ of $9 = 3$	$\frac{1}{3}$ of $24 = 8$
$3 \times 4 = 12$	$3 \times 9 = 27$	$\frac{1}{3}$ of $12 = 4$	$\frac{1}{3}$ of $27 = 9$

EXERCISE XXIII.

Copy and fill blanks:

1.	*2.*	*3.*
$\frac{2}{3}$ of $3 = (\ )$?	$\frac{2}{3}$ of $12 = (\ )$?	$\frac{1}{3}$ of $21 = (\ )$?
$\frac{1}{3}$ of $6 = (\ )$?	$\frac{1}{3}$ of $15 = (\ )$?	$\frac{2}{3}$ of $21 = (\ )$?
$\frac{2}{3}$ of $6 = (\ )$?	$3 \times 5 = (\ )$?	$3 \times 8 = (\ )$?
$3 \times 3 = (\ )$?	$\frac{2}{3}$ of $15 = (\ )$?	$\frac{1}{3}$ of $24 = (\ )$?
$\frac{1}{3}$ of $9 = (\ )$?	$3 \times 6 = (\ )$?	$\frac{2}{3}$ of $24 = (\ )$?
$\frac{2}{3}$ of $9 = (\ )$?	$\frac{1}{3}$ of $18 = (\ )$?	$3 \times 9 = (\ )$?
$\frac{2}{3}$ of $0 = (\ )$?	$\frac{2}{3}$ of $18 = (\ )$?	$\frac{1}{3}$ of $27 = (\ )$?
$3 \times 4 = (\ )$?	$3 \times 7 = (\ )$?	$\frac{2}{3}$ of $27 = (\ )$?

Call results quickly:

4.	*5.*	*6.*	*7.*
3×3	2×2	$\frac{1}{2}$ of 8	$\frac{2}{3}$ of 3
3×5	2×5	$\frac{1}{2}$ of 10	$\frac{2}{3}$ of 6
3×4	2×4	$\frac{1}{3}$ of 12	$\frac{2}{3}$ of 9
3×0	2×8	$\frac{1}{3}$ of 21	$\frac{2}{3}$ of 12
3×1	2×0	$\frac{1}{2}$ of 4	$\frac{2}{3}$ of 15
3×7	2×1	$\frac{1}{3}$ of 6	$\frac{1}{3}$ of 18
3×8	2×9	$\frac{1}{3}$ of 0	$\frac{1}{3}$ of 24
3×9	2×6	$\frac{1}{2}$ of 14	$\frac{1}{3}$ of 27

8. How many halves in a number? How many thirds?

9. $2\times\frac{1}{3}$ of a number $=(\frac{\ }{3})$ of the number?

NOTE.—Read "$(\frac{\ }{3})$," *how many thirds.*

10.	*11.*
$\frac{1}{2}+\frac{1}{2}=(\frac{\ }{2})$?	$(\frac{\ }{2})$ of an apple $=1$ apple?
$\frac{1}{3}+\frac{1}{3}=(\frac{\ }{3})$?	$(\frac{\ }{3})$ of an apple $=1$ apple?
$\frac{1}{3}+\frac{2}{3}=(\frac{\ }{3})$?	$(\frac{\ }{2})=1$?
$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=(\frac{\ }{3})$?	$(\frac{\ }{3})=1$?

Mixed drill—partly mental and partly written:

12. $25+3\times2=(\ \ )$?

Process: Think, "25 plus 6," looking upon "3×2" as 6, and as you think, write:

$$\begin{array}{r} 25 \\ \underline{\ 6} \end{array}$$

Then add.

13. $2\times9-\frac{1}{3}$ of $24=(\ \ )$?

Process: Looking upon "2×9" as 18, and "$\frac{1}{3}$ of 24" as 8, write:

$$\begin{array}{r} 18 \\ \underline{-\ 8} \end{array}$$

Then subtract. Remember that the *products* and *parts* must always be found before you add or subtract.

14. $2\times8+9=(\ \ )$?
15. $35-\frac{1}{3}$ of $12=(\ \ )$?
16. $27-\frac{2}{3}$ of $12=(\ \ )$?
17. $9+\frac{2}{3}$ of $15=(\ \ )$?
18. $7+\frac{2}{3}$ of $21=(\ \ )$?
19. $2\times9-3\times4=(\ \ )$?
20. $\frac{2}{3}$ of $27-3\times5=(\ \ )$?
21. $45-\frac{2}{3}$ of $9=(\ \ )$?
22. $\frac{1}{2}$ of $18+57=(\ \ )$?
23. $\frac{1}{3}$ of $24+42=(\ \ )$?
24. $\frac{2}{3}$ of $24-\frac{1}{3}$ of $18=(\ \ )$?
25. $\frac{2}{3}$ of $18-\frac{1}{2}$ of $18=(\ \ )$?
26. $\frac{2}{3}$ of $24+\frac{1}{2}$ of $16=(\ \ )$?
27. $\frac{1}{2}$ of $14+3\times9=(\ \ )$?
28. $3\times8-\frac{1}{3}$ of $15=(\ \ )$?
29. $2\times9-\frac{1}{3}$ of $9=(\ \ )$?

Call results quickly:

30.	*31.*	*32.*
$4=(\ \ )\times2$?	$12=(\ \ )\times4$?	$20=(\ \ )\times10$?
$6=(\ \ )\times3$?	$14=(\ \ )\times7$?	$21=(\ \ )\times\ 7$?
$6=(\ \ )\times2$?	$15=(\ \ )\times5$?	$24=(\ \ )\times\ 8$?
$8=(\ \ )\times4$?	$16=(\ \ )\times8$?	$12=(\ \ )\times\ 6$?
$9=(\ \ )\times3$?	$18=(\ \ )\times9$?	$27=(\ \ )\times\ 9$?
$10=(\ \ )\times5$?	$18=(\ \ )\times6$?	$30=(\ \ )\times10$?

In Ex. 30 read, "*4 equals how many times 2?*" etc.

33.	*34*	*35.*
$2=(\ \ )$ of 4?	$5=(\ \ )$ of 10?	$8=(\ \ )$ of 16?
$2=(\ \ )$ of 6?	$5=(\ \ )$ of 15?	$8=(\ \ )$ of 24?
$3=(\ \ )$ of 6?	$6=(\ \ )$ of 12?	$9=(\ \ )$ of 18?
$3=(\ \ )$ of 9?	$8=(\ \ )$ of 16?	$9=(\ \ )$ of 27?
$4=(\ \ )$ of 8?	$7=(\ \ )$ of 14?	$10=(\ \ )$ of 20?
$4=(\ \ )$ of 12?	$7=(\ \ )$ of 21?	$10=(\ \ )$ of 30?

In Ex. 33 read, "*2 equals what part of 4?*" etc.

24. Parts of a Problem.—The problems we now have to solve may each be divided into *two parts*, **Question** and **Basis.**

Problem: What will 3 tops cost, if 1 top costs 5¢?
Parts: (1) What will 3 tops cost? (Question.)
(2) 1 top costs 5¢. (Basis.)

We call one part the **Question,** because it asks a question. We call the other part the **Basis,** because from this part as a basis we must start to find out the answer to the problem.

EXAMPLES.

Teacher, explain these examples before giving the exercise:

1. What will 10 marbles cost at 2¢ each?

Parts: (1) What will 10 marbles cost? (Question.)
(2) 1 marble costs 2¢. (Basis.)

2. If I buy 2 books for $10, how many can I buy for $15?

Parts: (1) How many books can I buy for $15? (Question.)
(2) I buy 2 books for $10. (Basis.)

NOTE.—Write the question first, always.

3. There are 4 pecks in 1 bushel. How many pecks in 3 bushels?

Parts: (1) How many pecks in 3 bushels? (Question.)
(2) There are 4 pecks in 1 bushel. (Basis.)

4. If 2 cans of berries are worth 18¢, what is 1 can worth?

Parts: (1) What is 1 can worth? (Question.)
(2) 2 cans are worth 18¢. (Basis.)

5. Find the price of 3 hogs at $6 each.

Parts: (1) What is the price of 3 hogs? (Question.)
(2) $6 is the price of 1 hog. (Basis.)

EXERCISE XXIV.

Write the questions and bases as above:

1. What will 3 books cost at $4 each?

2. In 1 quart there are 2 pints. How many pints in 3 quarts?

3. What will 2 coats cost at $8 each?

4. In 3 nickels there are 15¢. How many cents in 1 nickel?

5. Find the cost of 3 oranges at 4¢ each.

25. More about the Parts of a Problem.—Each part of a problem may be written so as to contain the **Sign of Equality, =.** Take the following:

Parts: (1) What will 3 tops cost? (Question.)
(2) 1 top costs 5¢. (Basis.)

These may be written:

Parts: (1) Cost of 3 tops = ( )¢? (Question.)
(2) Cost of 1 top = 5¢. (Basis.)

These are called **Equations,** because the *sign of equality* is used in writing them. We must learn to use *equations* in solving problems.

EXAMPLES.

Teacher, explain these before giving the exercise:

1. What will 8 marbles cost, at 2¢ each?

Parts: (1) Cost of 8 marbles = ( )¢? (Question.)
(2) Cost of 1 marble = 2¢. (Basis.)

2. What will 3 pencils cost, if 2 pencils cost 10¢?

Parts: (1) Cost of 3 pencils = ( )¢? (Question.)
(2) Cost of 2 pencils = 10¢. (Basis.)

3. In 1 peck there are 8 quarts. How many quarts in 3 pecks?

Parts: (1) 3 pecks = ( ) quarts? (Question.)
(2) 1 peck = 8 quarts. (Basis.)

4. If a boy earns $15 in 3 weeks, how much is that per week?

Parts: (1) Earnings for 1 week = $( )? (Question.)
(2) Earnings for 3 weeks = $15. (Basis.)

5. Find the price of 3 apples, at 4¢ each?

Parts: (1) Price of 3 apples = ( )¢? (Question.)
(2) Price of 1 apple = 4¢. (Basis.)

EXERCISE XXV.

Write in equations the parts of the problems in Exercise XXIV.

26. Solving Problems.—First, write the question and basis, just as we have learned to do. Secondly, reason from what you have in the first part of the basis to **one**, then to the number required in the first part of the question.

EXAMPLES.

Teacher, be particular about explaining this set of examples:

1. What will 3 tops cost at 5¢ each?

Solution: (1) Cost of 3 tops = ( )¢? (Question.)
(2) Cost of 1 top = 5¢. (Basis.)
(3) Cost of 3 tops = 3 × 5¢ = 15¢, answer.

NOTE.—The first part of the basis is already *one;* we had only to go from *one* to *three* (from the "*cost of 1 top*" to the "*cost of 3 tops.*") 3 tops are how many times 1 top? Why did we get 3 × 5¢?

2. One bushel equals 4 pecks. How many pecks in 2 bushels?

Solution: (1) 2 bushels = ( ) pecks? (Question.)
(2) 1 bushel = 4 pecks. (Basis.)
(3) 2 bushels = 2 × 4 pecks = 8 pecks, answer.

NOTE.—Why did we get 2 × 4 pecks?

3. What is 1 pie worth, if I can get 2 pies for 16¢?

Solution: (1) Price of 1 pie = ( )¢? (Question.)
(2) Price of 2 pies = 16¢. (Basis.)
(3) Price of 1 pie = $\frac{1}{2}$ of 16¢ = 8¢, answer.

NOTE.—Here we had to go from *two* to *one* (from "price of 2 pies" to "price of 1 pie."). 1 is what part of 2? Why did we take $\frac{1}{2}$ of 16¢?

4. A boy earns $15 in 3 weeks. How much is that per week?

Solution: (1) Earnings for 1 week = $( )? (Question.)
(2) Earnings for 3 weeks = $15. (Basis.)
(3) Earnings for 1 week = $\frac{1}{3}$ of $15 = $5, answer.

NOTE.—One week is what part of 3 weeks? Why did we take $\frac{1}{3}$ of $15?

5. If 2 books are worth \$14, what are 3 books worth?

Solution: (1) Value of 3 books = \$( )? (Question.)
(2) Value of 2 books = \$14. (Basis.)
(3) Value of 1 book = $\frac{1}{2}$ of \$14 = \$7.
(4) Value of 3 books = 3 × \$7 = \$21, answer.

NOTE.—We reasoned first from *two* to *one*, then to *three*. 1 is what part of 2? 3 is how many times 1?

EXERCISE XXVI.

1. Three boys buy a melon for 15¢. How much should each pay?

Amount paid by 1 boy = ( )¢? (Question.)

2. What will 2 pencils cost, at 7¢ each?

3. If 2 calves are worth \$6, what is 1 calf worth?

4. Find the cost of 3 hogs, at \$9 each.

5. Find the weight of 3 loads of hay, of 2 tons each.

6. 24 yards of cloth was cut into 3 equal pieces. How many yards in each piece?

Length of 1 piece = ( ) yards? (Question.)

7. There are 8 quarts in 1 peck. How many quarts in 2 pecks?

2 pecks = ( ) quarts? (Question.)

8. In 3 yards there are 9 feet. How many feet in 1 yard?

9. A pole 21 feet long is sawed into 3 equal parts. How long is each part?

Length of 1 part = ( ) feet? (Question.)

10. I sold 3 calves for \$27. How much is that apiece?

11. If 3 mats cost \$9, what will 2 mats cost?

12. 2 bushels equal 8 pecks. How many pecks in 3 bushels?

27. Four Times and Fourths.—If we add together four numbers of the same size, the sum will be **Four Times** one of the numbers. Thus:

$$3+3+3+3=12;\ \text{then,}$$

$$\mathbf{4\times3=12}$$

If we divide a number into 4 equal parts, one of the parts will be **One-Fourth** (¼) of the number; two of the parts **Two-Fourths** (2/4); three parts **Three-Fourths** (¾); and all four parts **Four-Fourths** (4/4) of the number. Thus:

$$16=4+4+4+4;\ \text{then,}$$

$$\mathbf{¼\ of\ 16=4} \qquad \mathbf{¾\ of\ 16=12}$$
$$\mathbf{2/4\ of\ 16=8} \qquad \mathbf{4/4\ of\ 16=16}$$

$4\times0=0$	$4\times5=20$	¼ of 0=0	¼ of 20=5
$4\times1=4$	$4\times6=24$	¼ of 4=1	¼ of 24=6
$4\times2=8$	$4\times7=28$	¼ of 8=2	¼ of 28=7
$4\times3=12$	$4\times8=32$	¼ of 12=3	¼ of 32=8
$4\times4=16$	$4\times9=36$	¼ of 16=4	¼ of 36=9

EXERCISE XXVII.

Copy and fill blanks:

1.	*2.*	*3.*
4 × 0=( )?	4 × 1=( )?	4 × 2=( )?
¼ of 0=( )?	¼ of 4=( )?	¼ of 8=( )?
2/4 of 0=( )?	2/4 of 4=( )?	2/4 of 8=( )?
¾ of 0=( )?	¾ of 4=( )?	¾ of 8=( )?

4.	*5.*	*6.*
4 × 3=( )?	4 × 4=( )?	4 × 5=( )?
¼ of 12=( )?	¼ of 16=( )?	¼ of 20=( )?
2/4 of 12=( )?	2/4 of 16=( )?	2/4 of 20=( )?
¾ of 12=( )?	¾ of 16=( )?	¾ of 20=( )?

7.

$4 \times 6 = (\quad)$?
$\frac{1}{4}$ of $24 = (\quad)$?
$\frac{3}{4}$ of $24 = (\quad)$?
$4 \times 7 = (\quad)$?

8.

$\frac{1}{4}$ of $28 = (\quad)$?
$\frac{3}{4}$ of $28 = (\quad)$?
$4 \times 8 = (\quad)$?
$\frac{1}{4}$ of $32 = (\quad)$?

$\frac{3}{4}$ of $32 = (\quad)$?
$4 \times 9 = (\quad)$?
$\frac{1}{4}$ of $36 = (\quad)$?
$\frac{3}{4}$ of $36 = (\quad)$?

Call results quickly:

10.

4×6
4×8
4×5
4×9

11.

4×2
4×7
4×1
4×3

12.

$\frac{1}{4}$ of 8
$\frac{1}{4}$ of 16
$\frac{1}{4}$ of 12
$\frac{1}{4}$ of 28

13.

$\frac{1}{4}$ of 20
$\frac{1}{4}$ of 24
$\frac{1}{4}$ of 32
$\frac{1}{4}$ of 36

14.

$\frac{3}{4}$ of 4
$\frac{3}{4}$ of 8
$\frac{3}{4}$ of 12
$\frac{3}{4}$ of 16
$\frac{3}{4}$ of 20

15.

$\frac{3}{4}$ of 24
$\frac{3}{4}$ of 28
$\frac{3}{4}$ of 32
$\frac{3}{4}$ of 36
$\frac{3}{4}$ of 0

16.

$\frac{2}{4}$ of 0
$\frac{2}{4}$ of 4
$\frac{2}{4}$ of 8
$\frac{2}{4}$ of 12
$\frac{2}{4}$ of 16

17.

$\frac{2}{4}$ of 20
$\frac{2}{4}$ of 24
$\frac{2}{4}$ of 28
$\frac{2}{4}$ of 32
$\frac{2}{4}$ of 36

Copy and fill blanks:

18.

$(\frac{1}{4})$ of an apple = 1 apple?
$(\frac{1}{4})$ of an apple = $\frac{1}{2}$ of an apple?
$(\frac{1}{4}) = 1$?
$\frac{2}{4} = (\frac{1}{2})$?

19.

$\frac{1}{4} + \frac{1}{4} = (\frac{1}{4}) = (\frac{1}{2})$?
$\frac{4}{4} - \frac{1}{4} = (\frac{1}{4})$?
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = (\frac{1}{4}) = (\quad)$?
$\frac{3}{4} - \frac{1}{4} = (\frac{1}{4}) = (\frac{1}{2})$?

20.

$4 = (\quad) \times 1$?
$8 = 4 \times (\quad)$?
$12 = 4 \times (\quad)$?
$16 = (\quad) \times 4$?
$20 = (\quad) \times 5$?
$24 = 4 \times (\quad)$?
$28 = 4 \times (\quad)$?
$32 = (\quad) \times 8$?
$36 = (\quad) \times 9$?

21.

$1 = (\quad)$ of 4?
$2 = (\quad)$ of 8?
$3 = \frac{1}{4}$ of $(\quad)$?
$4 = \frac{1}{4}$ of $(\quad)$?
$5 = (\quad)$ of 20?
$6 = (\quad)$ of 24?
$7 = \frac{1}{4}$ of $(\quad)$?
$8 = \frac{1}{4}$ of $(\quad)$?
$9 = (\quad)$ of 36?

Give answers quickly:

22. $2\times1=1\times(\ )$? $3\times1=(\ )\times3$? $2\times(\ )=3\times2$?

23. $4\times1=(\ )\times4$? $(\ )\times2=2\times4$? $4\times3=3\times(\ )$?

Mixed drill:

24. $\frac{1}{4}$ of $12+\frac{1}{2}$ of $12+\frac{1}{3}$ of $12=(\ )$?

25. $2\times2+3\times2+4\times2=(\ )$?

26. $\frac{1}{4}$ of $16+\frac{1}{3}$ of $15+\frac{1}{2}$ of $14=(\ )$?

27. $\frac{1}{2}$ of $18-\frac{1}{4}$ of $16+\frac{1}{3}$ of $9=(\ )$?

28. $\frac{1}{4}$ of $24+\frac{1}{3}$ of $24-\frac{1}{2}$ of $16=(\ )$?

29. $\frac{1}{3}$ of $27-\frac{1}{4}$ of $28+\frac{1}{2}$ of $18=(\ )$?

30. $\frac{1}{4}$ of $36-\frac{1}{3}$ of $27+\frac{1}{3}$ of $21=(\ )$?

Problems; solve as in Article 26:

31. I bought 4 books, at \$6 each. How much did they cost me?

32. I bought 3 loads of corn for \$27. How much is that per load?

33. What cost 4 bottles of ink, at 5¢ each?

34. What cost 3 hats, at \$3 each?

35. What cost 4 calves, at \$7 each?

36. What is a suit of clothes worth, if 4 such suits are worth \$36?

37. Reduce 4 yards to feet. (1 yard=3 feet.)

38. Reduce 4 bushels to pecks. (1 bushel=4 pecks.)

39. Reduce 4 pecks to quarts. (1 peck=8 quarts.)

40. How many cents in 4 nickels? (1 nickel=5¢.)

41. If 4 pencils cost 16¢, what will 1 pencil cost?

42. If 4 plows cost \$36, what will 1 plow cost?

28. Five Times and Fifths.—If we add together five numbers of the same size, the sum will be **Five Times** one of the numbers.

If we divide a number into five equal parts, the parts will be **Fifths** of the number.

$5\times0=0$	$5\times5=25$	$\frac{1}{5}$ of $0=0$	$\frac{1}{5}$ of $25=5$
$5\times1=5$	$5\times6=30$	$\frac{1}{5}$ of $5=1$	$\frac{1}{5}$ of $30=6$
$5\times2=10$	$5\times7=35$	$\frac{1}{5}$ of $10=2$	$\frac{1}{5}$ of $35=7$
$5\times3=15$	$5\times8=40$	$\frac{1}{5}$ of $15=3$	$\frac{1}{5}$ of $40=8$
$5\times4=20$	$5\times9=45$	$\frac{1}{5}$ of $20=4$	$\frac{1}{5}$ of $45=9$

EXERCISE XXVIII.

Call results quickly:

1.	*2.*	*3.*	*4.*
5×3	4×3	$\frac{1}{5}$ of 0	$\frac{1}{4}$ of 12
5×7	4×5	$\frac{1}{5}$ of 15	$\frac{1}{4}$ of 20
5×4	4×8	$\frac{1}{5}$ of 20	$\frac{1}{4}$ of 28
5×0	3×7	$\frac{1}{5}$ of 40	$\frac{1}{4}$ of 32
5×8	4×7	$\frac{1}{5}$ of 30	$\frac{1}{4}$ of 24
5×5	4×9	$\frac{1}{5}$ of 45	$\frac{1}{3}$ of 21
5×9	4×6	$\frac{1}{5}$ of 35	$\frac{1}{3}$ of 24
5×1	3×5	$\frac{1}{5}$ of 25	$\frac{1}{3}$ of 18
5×2	3×9	$\frac{1}{5}$ of 10	$\frac{1}{3}$ of 27

Copy and fill blanks:

1.	*2.*	*3.*
$5\times1=(\quad)?$	$5\times2=(\quad)?$	$5\times3=(\quad)?$
$\frac{1}{5}$ of $5=(\quad)?$	$\frac{1}{5}$ of $10=(\quad)?$	$\frac{4}{5}$ of $15=(\quad)?$
$\frac{2}{5}$ of $5=(\quad)?$	$\frac{4}{5}$ of $10=(\quad)?$	$\frac{2}{5}$ of $15=(\quad)?$
$\frac{4}{5}$ of $5=(\quad)?$	$\frac{3}{5}$ of $10=(\quad)?$	$\frac{1}{5}$ of $15=(\quad)?$
$\frac{3}{5}$ of $5=(\quad)?$	$\frac{2}{5}$ of $10=(\quad)?$	$\frac{3}{5}$ of $15=(\quad)?$

4.	*5.*	*6.*
$5\times4=(\quad)?$	$5\times5=(\quad)?$	$5\times6=(\quad)?$
$\frac{1}{5}$ of $20=(\quad)?$	$\frac{4}{5}$ of $25=(\quad)?$	$\frac{3}{5}$ of $30=(\quad)?$
$\frac{3}{5}$ of $20=(\quad)?$	$\frac{2}{5}$ of $25=(\quad)?$	$\frac{1}{5}$ of $30=(\quad)?$
$\frac{4}{5}$ of $20=(\quad)?$	$\frac{3}{5}$ of $25=(\quad)?$	$\frac{4}{5}$ of $30=(\quad)?$
$\frac{2}{5}$ of $20=(\quad)?$	$\frac{1}{5}$ of $25=(\quad)?$	$\frac{2}{5}$ of $30=(\quad)?$

7.

$5 \times 7 = (\quad)$?
$\frac{3}{5}$ of $35 = (\quad)$?
$\frac{1}{5}$ of $35 = (\quad)$?
$\frac{4}{5}$ of $35 = (\quad)$?
$\frac{2}{5}$ of $35 = (\quad)$?

8.

$5 \times 8 = (\quad)$?
$\frac{1}{5}$ of $40 = (\quad)$?
$\frac{3}{5}$ of $40 = (\quad)$?
$\frac{4}{5}$ of $40 = (\quad)$?
$\frac{2}{5}$ of $40 = (\quad)$?

9.

$5 \times 9 = (\quad)$?
$\frac{3}{5}$ of $45 = (\quad)$?
$\frac{1}{5}$ of $45 = (\quad)$?
$\frac{4}{5}$ of $45 = (\quad)$?
$\frac{2}{5}$ of $45 = (\quad)$?

10.

$(\frac{}{5})$ of an apple = 1 apple?
$1 = (\frac{}{5})$?
$\frac{1}{5} + \frac{1}{5} = (\frac{}{5})$?
$\frac{1}{5} + \frac{2}{5} = (\frac{}{5})$?
$\frac{1}{5} + \frac{3}{5} = (\frac{}{5})$?

11.

$\frac{2}{5} + \frac{2}{5} = (\frac{}{5})$?
$\frac{3}{5} + \frac{2}{5} = (\frac{}{5}) = (\quad)$?
$\frac{3}{5} - \frac{2}{5} = (\frac{}{5})$?
$\frac{2}{5} - \frac{1}{5} = (\frac{}{5})$?
$\frac{4}{5} - \frac{2}{5} = (\frac{}{5})$?

12.

$5 = (\quad) \times 1$?
$10 = 5 \times (\quad)$?
$15 = (\quad) \times 3$?
$1 = \frac{1}{5}$ of $(\quad)$?
$3 = (\quad)$ of 15?
$20 = (\quad) \times 4$?
$2 = \frac{1}{5}$ of $(\quad)$?
$4 = (\quad)$ of 20?
$5 = \frac{1}{5}$ of $(\quad)$?

13.

$25 = (\quad) \times 5$?
$30 = 5 \times (\quad)$?
$6 = \frac{1}{5}$ of $(\quad)$?
$7 = (\quad)$ of 35?
$8 = \frac{1}{5}$ of $(\quad)$?
$40 = 5 \times (\quad)$?
$35 = 5 \times (\quad)$?
$45 = (\quad) \times 9$?
$9 = \frac{1}{5}$ of $(\quad)$?

14.

$4 \times 3 = (\quad) \times 4$?
$3 \times 2 = 2 \times (\quad)$?
$4 \times 1 = 1 \times (\quad)$?
$4 \times (\quad) = 0 \times 4$?
$5 \times (\quad) = 1 \times 5$?
$(\quad) \times 4 = 4 \times 5$?
$5 \times 3 = 3 \times (\quad)$?
$5 \times 2 = (\quad) \times 5$?
$5 \times (\quad) = 0 \times 5$?

Mixed drill:

15. $10 + \frac{1}{5}$ of $15 - \frac{1}{4}$ of $32 + \frac{1}{8}$ of $15 - \frac{1}{5}$ of $25 = (\quad)$?
16. $6 \times 3 + 5 \times 1 - \frac{1}{4}$ of $36 + \frac{1}{2}$ of $12 - 3 \times 2 = (\quad)$?
17. $4 \times 9 - 2 \times 4 - 3 \times 3 - \frac{1}{5}$ of $20 + \frac{1}{4}$ of $24 = (\quad)$?
18. $\frac{1}{5}$ of $35 - \frac{1}{5}$ of $25 + \frac{1}{5}$ of $45 - \frac{1}{5}$ of $40 = (\quad)$?
19. $\frac{1}{5}$ of $30 + \frac{1}{4}$ of $32 - \frac{1}{5}$ of $35 + \frac{1}{4}$ of $36 = (\quad)$?

29. Problems—Oral Solution.

EXAMPLES.

1. If 5 books cost \$10, what will 3 books cost?

Oral Solution: If 5 books cost \$10, 1 book will cost $\frac{1}{5}$ of \$10, or \$2. 3 books will cost $3 \times \$2$, or \$6.

2. Reduce 5 quarts to pints. (1 quart = 2 pints.)

Oral Solution: Since 1 quart equals 2 pints, 5 quarts equal 5×2 pints, or 10 pints.

NOTE.—The only difference between an *oral solution* and a *written solution* is, that the *oral solution* is expressed in *sentences;* the written solution is expressed in *equations.* The line of thought is exactly the same in both.

EXERCISE XXIX.

Give oral solutions; also, solve as in Article 26:

1. If 4 men cut 12 cords of wood, how many cords will 5 men cut?

Amount cut by 5 men = ( ) cords? (Question.)

2. If 5 books cost \$15, how much will 4 books cost?
3. What will 3 dozen apples cost, if 5 dozen sell for 35¢?
4. In 4 pecks there are 32 quarts. How many quarts in 5 pecks?
5. Reduce 5 bushels to pecks. (1 bushel = ( ) pecks?)
6. If a man works 40 hours in 5 days, how many hours does he work in 3 days?
7. Reduce 4 quarts to pints. (1 quart = ( ) pints?)
8. Reduce 3 pecks to quarts. (1 peck = ( ) quarts?)
9. If 5 pounds of meat sell for 35¢, how much is that per pound?
10. If 4 horses eat 20 bushels of oats, how much will 5 horses eat?

30. Six Times and Sixths.—If we add together six numbers of the same size, the sum will be **Six Times** one of the numbers.

If we divide a number into six equal parts, the parts will be **Sixths** of the number.

$6\times0=0$	$6\times5=30$	$\frac{1}{6}$ of $0=0$	$\frac{1}{6}$ of $30=5$
$6\times1=6$	$6\times6=36$	$\frac{1}{6}$ of $6=1$	$\frac{1}{6}$ of $36=6$
$6\times2=12$	$6\times7=42$	$\frac{1}{6}$ of $12=2$	$\frac{1}{6}$ of $42=7$
$6\times3=18$	$6\times8=48$	$\frac{1}{6}$ of $18=3$	$\frac{1}{6}$ of $48=8$
$6\times4=24$	$6\times9=54$	$\frac{1}{6}$ of $24=4$	$\frac{1}{6}$ of $54=9$

EXERCISE XXX.

Call results quickly:

1.	*2.*	*3.*	*4.*
$\frac{1}{6}$ of 0	$\frac{1}{6}$ of 48	4×9	3×7
$\frac{1}{5}$ of 0	$\frac{1}{5}$ of 45	5×9	5×4
$\frac{1}{4}$ of 0	$\frac{1}{3}$ of 24	6×9	6×8
$\frac{1}{3}$ of 0	$\frac{1}{6}$ of 42	3×9	4×7
$\frac{1}{2}$ of 0	$\frac{1}{5}$ of 35	5×8	6×5
$\frac{1}{6}$ of 36	$\frac{1}{4}$ of 36	5×5	4×5
$\frac{1}{5}$ of 30	$\frac{1}{6}$ of 24	6×6	3×5
$\frac{1}{4}$ of 32	$\frac{1}{6}$ of 18	4×4	6×3
$\frac{1}{3}$ of 27	$\frac{1}{5}$ of 25	6×7	5×3
$\frac{1}{6}$ of 54	$\frac{1}{6}$ of 30	5×7	6×9

Copy and fill blanks:

5.	*6.*	*7.*
$\frac{1}{6}$ of $6=(\ )$?	$\frac{1}{6}$ of $12=(\ )$?	$\frac{1}{6}$ of $18=(\ )$?
$\frac{2}{6}$ of $6=(\ )$?	$\frac{2}{6}$ of $12=(\ )$?	$\frac{2}{6}$ of $18=(\ )$?
$\frac{3}{6}$ of $6=(\ )$?	$\frac{3}{6}$ of $12=(\ )$?	$\frac{3}{6}$ of $18=(\ )$?
$\frac{4}{6}$ of $6=(\ )$?	$\frac{4}{6}$ of $12=(\ )$?	$\frac{4}{6}$ of $18=(\ )$?
$\frac{5}{6}$ of $6=(\ )$?	$\frac{5}{6}$ of $12=(\ )$?	$\frac{5}{6}$ of $18=(\ )$?

8.

$\frac{1}{6}$ of 24 = ( )?
$\frac{2}{6}$ of 24 = ( )?
$\frac{3}{6}$ of 24 = ( )?
$\frac{4}{6}$ of 24 = ( )?
$\frac{5}{6}$ of 24 = ( )?

9.

$\frac{1}{6}$ of 30 = ( )?
$\frac{2}{6}$ of 30 = ( )?
$\frac{3}{6}$ of 30 = ( )?
$\frac{4}{6}$ of 30 = ( )?
$\frac{5}{6}$ of 30 = ( )?

10.

$\frac{1}{6}$ of 36 = ( )?
$\frac{2}{6}$ of 36 = ( )?
$\frac{3}{6}$ of 36 = ( )?
$\frac{4}{6}$ of 36 = ( )?
$\frac{5}{6}$ of 36 = ( )?

11.

$\frac{1}{6}$ of 42 = ( )?
$\frac{2}{6}$ of 42 = ( )?
$\frac{3}{6}$ of 42 = ( )?
$\frac{4}{6}$ of 42 = ( )?
$\frac{5}{6}$ of 42 = ( )?

12.

$\frac{1}{6}$ of 48 = ( )?
$\frac{2}{6}$ of 48 = ( )?
$\frac{3}{6}$ of 48 = ( )?
$\frac{4}{6}$ of 48 = ( )?
$\frac{5}{6}$ of 48 = ( )?

13.

$\frac{1}{6}$ of 54 = ( )?
$\frac{2}{6}$ of 54 = ( )?
$\frac{3}{6}$ of 54 = ( )?
$\frac{4}{6}$ of 54 = ( )?
$\frac{5}{6}$ of 54 = ( )?

14.

$(\frac{1}{6})$ of an apple = 1 apple?
$(\frac{1}{6})$ of an apple = $\frac{1}{2}$ of an apple?
$(\frac{1}{6})$ of an apple = $\frac{1}{3}$ of an apple?
$(\frac{1}{6})$ of an apple = $\frac{2}{3}$ of an apple?

15.

$(\frac{1}{6}) = 1$?
$(\frac{1}{6}) = \frac{1}{2}$?
$(\frac{1}{6}) = \frac{1}{3}$?
$(\frac{1}{6}) = \frac{2}{3}$?

16.

$6 \times 5 = (\ ) \times 6$?
$6 \times (\ ) = 4 \times 6$?
$6 \times 3 = 3 \times (\ )$?
$(\ ) \times 2 = 2 \times 6$?

NOTE.—Teacher, explain No. 14 with objects, if necessary. Divide an apple (or something) into sixths, halves, and thirds, and show these relations.

17.

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + = (\frac{1}{6}) = (\frac{1}{2})$?
$\frac{1}{6} + \frac{1}{6} = (\frac{1}{6}) = (\frac{1}{3})$?
$\frac{1}{6} + \frac{3}{6} = (\frac{1}{6}) = (\frac{1}{3})$?
$\frac{2}{6} + \frac{3}{6} = (\frac{1}{6})$?

18.

$\frac{6}{6} - \frac{3}{6} = (\frac{1}{6}) = (\frac{1}{2})$?
$\frac{5}{6} - \frac{1}{6} = (\frac{1}{6}) = (\frac{1}{3})$?
$\frac{4}{6} - \frac{2}{6} = (\frac{1}{6}) = (\frac{1}{6})$?
$\frac{4}{6} - \frac{2}{6} + \frac{3}{6} = (\frac{1}{6})$?

Mixed drill:

19. $\frac{1}{6}$ of $42 + \frac{1}{5}$ of $35 + \frac{1}{4}$ of $28 + \frac{1}{3}$ of $21 = (\ )$?

20. $\frac{1}{5}$ of $30 - \frac{1}{6}$ of $30 + \frac{1}{4}$ of $36 - \frac{1}{5}$ of $35 = (\ )$?

21. $\frac{2}{3}$ of $12 - \frac{1}{6}$ of $36 + \frac{1}{6}$ of $24 + 4 \times 6 = (\ )$?

22. $\frac{3}{5}$ of $25 - \frac{1}{4}$ of $36 + \frac{1}{5}$ of $40 - \frac{1}{4}$ of $28 = (\ )$?

23. $6 \times 9 - \frac{1}{5}$ of $45 - \frac{1}{4}$ of $36 - \frac{1}{3}$ of $27 = (\ )$?

24. $6 \times 8 - \frac{1}{6}$ of $42 - \frac{1}{5}$ of $30 - \frac{1}{6}$ of $18 = (\ )$?

31. Problems—Arranging the Question.

EXAMPLES.

1. If 6 books cost \$12, what cost 4 books?

Solution: (1) Cost of 4 books = \$( )? (Question.)
(2) Cost of 6 books = \$12. (Basis.)
(3) Cost of 1 book = $\frac{1}{6}$ of \$12 = \$2.
(4) Cost of 4 books = 4 × \$2 = \$8. (Ans.)

2. If 12 books cost \$6, how many books can I buy for \$4?

Solution: (1) \$4 = cost of ( ) books? (Question.)
(2) \$6 = cost of 12 books. (Basis.)
(3) \$1 = cost of $\frac{1}{6}$ of 12 books, or 2 books.
(4) \$4 = cost of 4 × 2 books, or 8 books. (Ans.)

NOTE.—In the first solution, what did we want in the *answer*, *dollars* or *books?* What did we put on the right-hand in the *equations*, *dollars* or *books?*

In the second solution, what did we want in the *answer*, *dollars* or *books?* What did we put on the right-hand in the *equations*, *dollars* or *books?* The *blank term* in the *question* always shows what is wanted in the *answer*.

Remember, that you should always write the question with its blank term on the right of the sign of equality, =; and the basis should be written in the form of the question.

3. If \$6 will buy 24 yards of cloth, how many yards can be bought for \$5?

Oral Solution: If \$6 will buy 24 yards, \$1 will buy $\frac{1}{6}$ of 24 yards, or 4 yards. \$5 will buy 5 × 4 yards, or 20 yards.

EXERCISE XXXI.

For written and oral work:

1. Find the cost of 6 tablets of paper, at 8¢ each

2. If 6 men cut 54 cords of wood, how many cords can 2 men cut in the same time?

3. If \$6 buy 36 primers, how many will \$4 buy?

4. If \$24 will buy 6 yards of cloth, how many yards will \$48 buy?

5. How many cents will buy 3 pounds of rice, if 5 pounds cost 45¢?

6. If 3 dozen eggs sell for 24¢, what will 6 dozen sell for?

7. 48 apples are divided equally among 6 children. How many does each one get?

1 share = ( ) apples? (Question.)

8. A boy spends 40¢ in 5 days. At that rate, how much will he spend in 6 days?

Amount spent in 6 days = ( )¢? (Question.)

9. A man earns \$36 in a week (6 days). How much is that per day?

Earnings for 1 day = \$( )? (Question.)

10. A lady sews a seam 24 inches long in 4 minutes. How far can she sew in 6 minutes?

Distance sewed in 6 min. = ( ) in.? (Question.)

32. Seven Times and Sevenths.—If we add together seven numbers of the same size, the sum will be **Seven Times** one of the numbers.

If we divide a number into seven equal parts, the parts will be **Sevenths** of the number.

$7\times0=0$	$7\times5=35$	$\frac{1}{7}$ of $0=0$	$\frac{1}{7}$ of $35=5$
$7\times1=7$	$7\times6=42$	$\frac{1}{7}$ of $7=1$	$\frac{1}{7}$ of $42=6$
$7\times2=14$	$7\times7=49$	$\frac{1}{7}$ of $14=2$	$\frac{1}{7}$ of $49=7$
$7\times3=21$	$7\times8=56$	$\frac{1}{7}$ of $21=3$	$\frac{1}{7}$ of $56=8$
$7\times4=28$	$7\times9=63$	$\frac{1}{7}$ of $28=4$	$\frac{1}{7}$ of $63=9$

EXERCISE XXXII.

Call results quickly:

1.	*2.*	*3.*	*4.*
$\frac{1}{7}$ of 7	$\frac{1}{5}$ of 30	7×2	5×4
$\frac{1}{7}$ of 14	$\frac{1}{6}$ of 36	7×9	5×8
$\frac{1}{7}$ of 42	$\frac{1}{7}$ of 35	7×8	5×9
$\frac{1}{7}$ of 63	$\frac{1}{7}$ of 28	7×6	5×7
$\frac{1}{7}$ of 56	$\frac{1}{6}$ of 36	7×7	5×6
$\frac{1}{6}$ of 54	$\frac{1}{7}$ of 49	6×6	5×0
$\frac{1}{6}$ of 42	$\frac{1}{7}$ of 0	6×7	4×7
$\frac{1}{7}$ of 21	$\frac{1}{6}$ of 24	6×9	4×8
$\frac{1}{5}$ of 45	$\frac{1}{4}$ of 32	6×8	4×4
$\frac{1}{6}$ of 48	$\frac{1}{4}$ of 36	6×5	4×9

Copy and fill blanks; afterwards use orally:

5.	*6.*	*7.*
$\frac{1}{7}$ of 7=( )?	$\frac{1}{7}$ of 14=( )?	$\frac{1}{7}$ of 21=( )?
$\frac{2}{7}$ of 7=( )?	$\frac{2}{7}$ of 14=( )?	$\frac{2}{7}$ of 21=( )?
$\frac{3}{7}$ of 7=( )?	$\frac{3}{7}$ of 14=( )?	$\frac{3}{7}$ of 21=( )?
$\frac{4}{7}$ of 7=( )?	$\frac{4}{7}$ of 14=( )?	$\frac{4}{7}$ of 21=( )?
$\frac{5}{7}$ of 7=( )?	$\frac{5}{7}$ of 14=( )?	$\frac{5}{7}$ of 21=( )?
$\frac{6}{7}$ of 7=( )?	$\frac{6}{7}$ of 14=( )?	$\frac{6}{7}$ of 21=( )?

8.	*9.*	*10.*
$\frac{1}{7}$ of 28=( )?	$\frac{1}{7}$ of 35=( )?	$\frac{1}{7}$ of 42=( )?
$\frac{2}{7}$ of 28=( )?	$\frac{2}{7}$ of 35=( )?	$\frac{2}{7}$ of 42=( )?
$\frac{3}{7}$ of 28=( )?	$\frac{3}{7}$ of 35=( )?	$\frac{3}{7}$ of 42=( )?
$\frac{4}{7}$ of 28=( )?	$\frac{4}{7}$ of 35=( )?	$\frac{4}{7}$ of 42=( )?
$\frac{5}{7}$ of 28=( )?	$\frac{5}{7}$ of 35=( )?	$\frac{5}{7}$ of 42=( )?
$\frac{6}{7}$ of 28=( )?	$\frac{6}{7}$ of 35=( )?	$\frac{6}{7}$ of 42=( )?

11.	*12.*	*13.*
$\frac{1}{7}$ of 49=( )?	$\frac{1}{7}$ of 56=( )?	$\frac{1}{7}$ of 63=( )?
$\frac{2}{7}$ of 49=( )?	$\frac{2}{7}$ of 56=( )?	$\frac{2}{7}$ of 63=( )?
$\frac{3}{7}$ of 49=( )?	$\frac{3}{7}$ of 56=( )?	$\frac{3}{7}$ of 63=( )?
$\frac{4}{7}$ of 49=( )?	$\frac{4}{7}$ of 56=( )?	$\frac{4}{7}$ of 63=( )?
$\frac{5}{7}$ of 49=( )?	$\frac{5}{7}$ of 56=( )?	$\frac{5}{7}$ of 63=( )?
$\frac{6}{7}$ of 49=( )?	$\frac{6}{7}$ of 56=( )?	$\frac{6}{7}$ of 63=( )?

14.

$(\frac{}{7})=1$?
$\frac{1}{7}+\frac{2}{7}=(\frac{}{7})$?
$\frac{2}{7}-\frac{1}{7}=(\frac{}{7})$?
$\frac{3}{7}+\frac{1}{7}=(\frac{}{7})$?

15.

$\frac{3}{7}+\frac{2}{7}=(\frac{}{7})$?
$\frac{5}{7}-\frac{2}{7}=(\frac{}{7})$?
$\frac{4}{7}+\frac{3}{7}=(\frac{}{7})=(\quad)$?
$\frac{6}{7}-\frac{4}{7}=(\frac{}{7})$?

16.

$\frac{7}{7}-\frac{5}{7}+\frac{2}{7}=(\frac{}{7})$?
$\frac{5}{7}+\frac{2}{7}-\frac{6}{7}=(\frac{}{7})$?
$\frac{4}{7}-\frac{2}{7}+\frac{3}{7}=(\frac{}{7})$?
$\frac{5}{7}-\frac{1}{7}+\frac{2}{7}=(\frac{}{7})$?

17.

$14=(\quad)\times 2$?
$21=7\times(\quad)$?
$28=(\quad)\times 4$?
$35=(\quad)\times 5$?

18.

$42=(\quad)\times 6$?
$49=7\times(\quad)$?
$56=7\times(\quad)$?
$63=(\quad)\times 9$?

19.

$2=(\quad)$ of 14?
$3=(\quad)$ of 21?
$4=\frac{1}{7}$ of $(\quad)$?
$5=\frac{1}{7}$ of $(\quad)$?

20.

$6=\frac{1}{7}$ of $(\quad)$?
$7=(\quad)$ of 49?
$8=(\quad)$ of 56?
$9=\frac{1}{7}$ of $(\quad)$?

21.

$7\times 3=(\quad)\times 7$?
$7\times(\quad)=4\times 7$?
$7\times 5=5\times(\quad)$?
$7\times 6=6\times(\quad)$?

Mixed drill:

22. $7\times 8-\frac{1}{5}$ of $30-\frac{1}{7}$ of $49-\frac{1}{6}$ of $36=(\quad)$?
23. $7\times 7-\frac{1}{7}$ of $49-\frac{1}{6}$ of $54+\frac{1}{5}$ of $35=(\quad)$?
24. $7\times 9-\frac{1}{4}$ of $24-\frac{1}{5}$ of $35-\frac{1}{2}$ of $18=(\quad)$?
25. $7\times 6-\frac{1}{4}$ of $36-\frac{1}{6}$ of $36-\frac{1}{7}$ of $49=(\quad)$?
26. $\frac{1}{5}$ of $25+\frac{1}{6}$ of $54-\frac{1}{7}$ of $63+\frac{1}{4}$ of $28=(\quad)$?
27. $\frac{1}{2}$ of $18-\frac{1}{3}$ of $21+\frac{1}{7}$ of $42-\frac{1}{6}$ of $48=(\quad)$?

Give oral solution:

28. What will 7 apple trees cost, at 9¢ each?

29. If \$7 will buy 28 books, at that rate what will \$5 buy?

30. If 6 hats are worth \$24, what are 7 such hats worth?

31. If \$7 are paid for 35 melons, how many melons will \$5 buy?

32. What will 7 suits of clothes cost, at \$8 each?

33. If 7 pounds of sugar are worth 49¢, what will 6 pounds cost?

34. What will 7 hogs sell for, if 4 such hogs sell for $36?

35. If $6 will pay for 2 pigs, how much will 7 such pigs cost?

33. Eight Times and Eighths.—If we add together eight numbers of the same size, the sum will be **Eight Times** one of the numbers.

If we divide a number into eight equal parts, the parts will be **Eighths** of the number.

8×0= 0	**8×5=40**	**⅛ of 0=0**	**⅛ of 40=5**
8×1= 8	**8×6=48**	**⅛ of 8=1**	**⅛ of 48=6**
8×2=16	**8×7=56**	**⅛ of 16=2**	**⅛ of 56=7**
8×3=24	**8×8=64**	**⅛ of 24=3**	**⅛ of 64=8**
8×4=32	**8×9=72**	**⅛ of 32=4**	**⅛ of 72=9**

EXERCISE XXXIII.

Call results quickly:

1.	*2.*	*3.*	*4.*
8×0	7×8	$\frac{1}{7}$ of 63	$\frac{1}{5}$ of 45
8×5	7×7	$\frac{1}{8}$ of 72	$\frac{1}{4}$ of 36
8×9	7×9	$\frac{1}{6}$ of 42	$\frac{1}{6}$ of 48
8×6	6×9	$\frac{1}{8}$ of 56	$\frac{1}{7}$ of 56
8×7	5×9	$\frac{1}{7}$ of 49	$\frac{1}{8}$ of 32
8×3	6×8	$\frac{1}{8}$ of 64	$\frac{1}{7}$ of 42
8×8	7×5	$\frac{1}{6}$ of 36	$\frac{1}{8}$ of 24
8×4	6×6	$\frac{1}{8}$ of 48	$\frac{1}{7}$ of 28

Copy and fill blanks; afterwards use orally:

5.	*6.*	*7.*
$(\frac{}{8})=1$?	$\frac{1}{8}+\frac{3}{8}=(\frac{}{8})=(\frac{}{2})$?	$\frac{7}{8}-\frac{5}{8}=(\frac{}{8})=(\frac{}{4})$?
$(\frac{}{8})=\frac{1}{2}=(\frac{}{4})$?	$\frac{3}{8}+\frac{3}{8}=(\frac{}{8})=(\frac{}{4})$?	$\frac{5}{8}-\frac{1}{8}=(\frac{}{8})=(\frac{}{2})$?
$(\frac{}{8})=\frac{1}{4}$?	$\frac{5}{8}+\frac{2}{8}=(\frac{}{8})$?	$\frac{6}{8}-\frac{3}{8}=(\frac{}{8})$?
$(\frac{}{8})=\frac{3}{4}$?	$\frac{2}{8}+\frac{2}{8}+\frac{1}{8}=(\frac{}{8})$?	$\frac{7}{8}-\frac{4}{8}+\frac{2}{8}=(\frac{}{8})$?

8.

$\frac{1}{8}$ of 8=( )?
$\frac{2}{8}$ of 8=( )?
$\frac{3}{8}$ of 8=( )?
$\frac{4}{8}$ of 8=( )?
$\frac{5}{8}$ of 8=( )?
$\frac{6}{8}$ of 8=( )?
$\frac{7}{8}$ of 8=( )?

9.

$\frac{1}{8}$ of 16=( )?
$\frac{2}{8}$ of 16=( )?
$\frac{3}{8}$ of 16=( )?
$\frac{4}{8}$ of 16=( )?
$\frac{5}{8}$ of 16=( )?
$\frac{6}{8}$ of 16=( )?
$\frac{7}{8}$ of 16=( )?

10.

$\frac{1}{8}$ of 24=( )?
$\frac{2}{8}$ of 24=( )?
$\frac{3}{8}$ of 24=( )?
$\frac{4}{8}$ of 24=( )?
$\frac{5}{8}$ of 24=( )?
$\frac{6}{8}$ of 24=( )?
$\frac{7}{8}$ of 24=( )?

11.

$\frac{1}{8}$ of 32=( )?
$\frac{2}{8}$ of 32=( )?
$\frac{3}{8}$ of 32=( )?
$\frac{4}{8}$ of 32=( )?
$\frac{5}{8}$ of 32=( )?
$\frac{6}{8}$ of 32=( )?
$\frac{7}{8}$ of 32=( )?

12.

$\frac{1}{8}$ of 40=( )?
$\frac{2}{8}$ of 40=( )?
$\frac{3}{8}$ of 40=( )?
$\frac{4}{8}$ of 40=( )?
$\frac{5}{8}$ of 40=( )?
$\frac{6}{8}$ of 40=( )?
$\frac{7}{8}$ of 40=( )?

13.

$\frac{1}{8}$ of 48=( )?
$\frac{2}{8}$ of 48=( )?
$\frac{3}{8}$ of 48=( )?
$\frac{4}{8}$ of 48=( )?
$\frac{5}{8}$ of 48=( )?
$\frac{6}{8}$ of 48=( )?
$\frac{7}{8}$ of 48=( )?

14.

$\frac{1}{8}$ of 56=( )?
$\frac{2}{8}$ of 56=( )?
$\frac{3}{8}$ of 56=( )?
$\frac{4}{8}$ of 56=( )?
$\frac{5}{8}$ of 56=( )?
$\frac{6}{8}$ of 56=( )?
$\frac{7}{8}$ of 56=( )?

15.

$\frac{1}{8}$ of 64=( )?
$\frac{2}{8}$ of 64=( )?
$\frac{3}{8}$ of 64=( )?
$\frac{4}{8}$ of 64=( )?
$\frac{5}{8}$ of 64=( )?
$\frac{6}{8}$ of 64=( )?
$\frac{7}{8}$ of 64=( )?

16.

$\frac{1}{8}$ of 72=( )?
$\frac{2}{8}$ of 72=( )?
$\frac{3}{8}$ of 72=( )?
$\frac{4}{8}$ of 72=( )?
$\frac{5}{8}$ of 72=( )?
$\frac{6}{8}$ of 72=( )?
$\frac{7}{8}$ of 72=( )?

17.

$1=\frac{1}{8}$ of ( )?
$2=\frac{1}{8}$ of ( )?
$3=\frac{1}{8}$ of ( )?
$4=\frac{1}{8}$ of ( )?
$5=\frac{1}{8}$ of ( )?
$6=\frac{1}{8}$ of ( )?
$7=\frac{1}{8}$ of ( )?
$8=\frac{1}{8}$ of ( )?
$9=\frac{1}{8}$ of ( )?

18.

8=8×( )?
16=8×( )?
24=8×( )?
32=8×( )?
40=8×( )?
48=8×( )?
56=8×( )?
64=8×( )?
72=8×( )?

19.

8×1=1×( )?
8×2=2×( )?
8×3=3×( )?
8×4=4×( )?
8×5=5×( )?
8×6=6×( )?
8×7=7×( )?
8×8=8×( )?
8×0=( )×8?

Mixed drill:

20. $8\times8-\frac{1}{8}$ of $56-\frac{1}{7}$ of $63-\frac{1}{5}$ of $40=(\quad)$?

21. $8\times9-\frac{1}{7}$ of $56-\frac{1}{6}$ of $48-\frac{1}{4}$ of $36=(\quad)$?

22. $8\times6-\frac{1}{5}$ of $35-\frac{1}{6}$ of $42-\frac{1}{7}$ of $49=(\quad)$?

23. $8\times7-\frac{1}{8}$ of $72-\frac{1}{7}$ of $35-\frac{1}{5}$ of $45=(\quad)$?

24. $8\times5-\frac{1}{7}$ of $63-\frac{1}{4}$ of $36-\frac{1}{6}$ of $36=(\quad)$?

Write solutions:

25. I sold 8 melons at 9¢ each. How much did I get for them?

26. A man can earn \$72 in 8 weeks. How much can he earn in 7 weeks?

27. A horse travels 32 miles in 4 hours. At that rate, how far can he travel in 8 hours?

Distance traveled in 8 hours = ( ) miles? (Question.)

28. If 3 tables are worth \$18, what will 8 such tables cost?

29. If \$8 will buy 24 books, how many can be bought for \$5?

30. If 6 bushels of apples cost \$3, how many bushels can I buy for \$8?

31. A company of people eat 24 loaves of bread in 3 days. How many loaves will they eat in 8 days?

Amount eaten in 3 da. = 24 loaves. (Basis.)

32. If 4 turkeys weigh 28 pounds, at that rate what will 8 turkeys weigh?

34. Nine Times and Ninths.—If we add together nine numbers of the same size, the sum will be **Nine Times** one of the numbers.

If we divide a number into nine equal parts, the parts will be **Ninths** of the number.

$9\times0=0$	$9\times5=45$	$\frac{1}{9}$ of $0=0$	$\frac{1}{9}$ of $45=5$
$9\times1=9$	$9\times6=54$	$\frac{1}{9}$ of $9=1$	$\frac{1}{9}$ of $54=6$
$9\times2=18$	$9\times7=63$	$\frac{1}{9}$ of $18=2$	$\frac{1}{9}$ of $63=7$
$9\times3=27$	$9\times8=72$	$\frac{1}{9}$ of $27=3$	$\frac{1}{9}$ of $72=8$
$9\times4=36$	$9\times9=81$	$\frac{1}{9}$ of $36=4$	$\frac{1}{9}$ of $81=9$

EXERCISE XXXIV.

Call results quickly:

1.	*2.*	*3.*	*4.*
9×0	8×9	$\frac{1}{9}$ of 0	$\frac{1}{9}$ of 36
9×3	8×7	$\frac{1}{9}$ of 72	$\frac{1}{8}$ of 40
9×5	8×6	$\frac{1}{8}$ of 64	$\frac{1}{9}$ of 45
9×8	8×8	$\frac{1}{9}$ of 81	$\frac{1}{7}$ of 21
9×6	7×8	$\frac{1}{7}$ of 49	$\frac{1}{9}$ of 27
9×4	6×9	$\frac{1}{9}$ of 63	$\frac{1}{8}$ of 48
9×2	7×9	$\frac{1}{6}$ of 54	$\frac{1}{4}$ of 36
9×9	7×6	$\frac{1}{7}$ of 56	$\frac{1}{5}$ of 45
9×7	7×7	$\frac{1}{8}$ of 72	$\frac{1}{9}$ of 54

Copy and fill blanks:

5.	*6.*	*7.*
$\frac{1}{9}$ of $9=(\ )$?	$\frac{1}{9}$ of $18=(\ )$?	$\frac{1}{9}$ of $27=(\ )$?
$\frac{2}{9}$ of $9=(\ )$?	$\frac{2}{9}$ of $18=(\ )$?	$\frac{2}{9}$ of $27=(\ )$?
$\frac{3}{9}$ of $9=(\ )$?	$\frac{3}{9}$ of $18=(\ )$?	$\frac{3}{9}$ of $27=(\ )$?
$\frac{4}{9}$ of $9=(\ )$?	$\frac{4}{9}$ of $18=(\ )$?	$\frac{4}{9}$ of $27=(\ )$?
$\frac{5}{9}$ of $9=(\ )$?	$\frac{5}{9}$ of $18=(\ )$?	$\frac{5}{9}$ of $27=(\ )$?
$\frac{6}{9}$ of $9=(\ )$?	$\frac{6}{9}$ of $18=(\ )$?	$\frac{6}{9}$ of $27=(\ )$?
$\frac{7}{9}$ of $9=(\ )$?	$\frac{7}{9}$ of $18=(\ )$?	$\frac{7}{9}$ of $27=(\ )$?
$\frac{8}{9}$ of $9=(\ )$?	$\frac{8}{9}$ of $18=(\ )$?	$\frac{8}{9}$ of $27=(\ )$?

8.	*9.*	*10.*
$\frac{1}{9}$ of $36=(\ )$?	$\frac{1}{9}$ of $45=(\ )$?	$\frac{1}{9}$ of $54=(\ )$?
$\frac{2}{9}$ of $36=(\ )$?	$\frac{2}{9}$ of $45=(\ )$?	$\frac{2}{9}$ of $54=(\ )$?
$\frac{3}{9}$ of $36=(\ )$?	$\frac{3}{9}$ of $45=(\ )$?	$\frac{3}{9}$ of $54=(\ )$?
$\frac{4}{9}$ of $36=(\ )$?	$\frac{4}{9}$ of $45=(\ )$?	$\frac{4}{9}$ of $54=(\ )$?
$\frac{5}{9}$ of $36=(\ )$?	$\frac{5}{9}$ of $45=(\ )$?	$\frac{5}{9}$ of $54=(\ )$?
$\frac{6}{9}$ of $36=(\ )$?	$\frac{6}{9}$ of $45=(\ )$?	$\frac{6}{9}$ of $54=(\ )$?
$\frac{7}{9}$ of $36=(\ )$?	$\frac{7}{9}$ of $45=(\ )$?	$\frac{7}{9}$ of $54=(\ )$?
$\frac{8}{9}$ of $36=(\ )$?	$\frac{8}{9}$ of $45=(\ )$?	$\frac{8}{9}$ of $54=(\ )$?

11.

$\frac{1}{9}$ of 63=( )?
$\frac{2}{9}$ of 63=( )?
$\frac{3}{9}$ of 63=( )?
$\frac{4}{9}$ of 63=( )?
$\frac{5}{9}$ of 63=( )?
$\frac{6}{9}$ of 63=( )?
$\frac{7}{9}$ of 63=( )?
$\frac{8}{9}$ of 63=( )?

12.

$\frac{1}{9}$ of 72=( )?
$\frac{2}{9}$ of 72=( )?
$\frac{3}{9}$ of 72=( )?
$\frac{4}{9}$ of 72=( )?
$\frac{5}{9}$ of 72=( )?
$\frac{6}{9}$ of 72=( )?
$\frac{7}{9}$ of 72=( )?
$\frac{8}{9}$ of 72=( )?

13.

$\frac{1}{9}$ of 81=( )?
$\frac{2}{9}$ of 81=( )?
$\frac{3}{9}$ of 81=( )?
$\frac{4}{9}$ of 81=( )?
$\frac{5}{9}$ of 81=( )?
$\frac{6}{9}$ of 81=( )?
$\frac{7}{9}$ of 81=( )?
$\frac{8}{9}$ of 81=( )?

14.

$(\frac{}{9})=1$
$(\frac{}{9})=\frac{1}{8}$
$(\frac{}{9})=\frac{2}{8}$

15.

$\frac{5}{9}+\frac{3}{9}=(\frac{}{9})$?
$\frac{7}{9}-\frac{4}{9}=(\frac{}{9})=(\frac{}{3})$?
$\frac{5}{9}+\frac{1}{9}=(\frac{}{9})=$( )?

16.

$\frac{8}{9}-\frac{2}{9}=(\frac{}{9})=(\frac{}{3})$?
$\frac{7}{9}-\frac{5}{9}+\frac{1}{9}=(\frac{}{3})$?
$\frac{8}{9}+\frac{5}{9}-\frac{7}{9}=(\frac{}{9})$?

17.

1=( ) of 9?
2=( ) of 18?
3=$\frac{1}{9}$ of ( )?
4=$\frac{1}{9}$ of ( )?
5=$\frac{1}{9}$ of ( )?
6=$\frac{1}{9}$ of ( )?
7=$\frac{1}{9}$ of ( )?
8=$\frac{1}{9}$ of ( )?
9=$\frac{1}{9}$ of ( )?

18.

9=( )×1?
18=9×( )?
27=9×( )?
36=9×( )?
45=( )×5?
54=( )×6?
63=9×( )?
72=9×( )?
81=( )×9?

19.

9×1=( )×9?
9×2=2×( )?
9×3=3×( )?
9×4=4×( )?
9×5=( )×9?
9×6=( )×9?
9×7=7×( )?
9×8=8×( )?
9×9=( )×9?

Mixed drill:

20. $\frac{1}{9}$ of 72+$\frac{1}{8}$ of 64−$\frac{1}{7}$ of 63+$\frac{1}{6}$ of 48=( )?

21. $\frac{1}{7}$ of 42−$\frac{1}{5}$ of 36+$\frac{1}{5}$ of 45−$\frac{1}{8}$ of 56=( )?

22. $\frac{1}{8}$ of 72−$\frac{1}{7}$ of 49+$\frac{1}{6}$ of 54−$\frac{1}{7}$ of 63=( )?

23. 9×8−$\frac{1}{9}$ of 81−$\frac{1}{7}$ of 35−$\frac{1}{8}$ of 56=( )?

Write solutions:

24. If 7 melons cost 56¢, what will 9 melons cost?

25. If 6 yards of calico cost 36¢, what will 8 yards cost?

26. If 8 hogs cost $72, what will 6 hogs cost?

27. How many cents in 9 nickels? (1 nickel = ( )¢?)

28. Reduce 9 pecks to quarts. (1 peck = ( ) quarts?)

29. If 9 men build 63 rods of fence, how many rods could 5 men build in the same time?

30. Reduce 9 square yards to square feet. (1 square yard = 9 square feet.)

31. Reduce 9 bushels to pecks. (1 bushel = ( ) pecks?)

32. I paid $7 for 35 books. How many such books can I get for $9?

33. What will 9 dozen eggs bring, at 8¢ per dozen?

34. If 8 books cost $16, what will 9 books cost?

V. DIVISION TO EIGHTY-ONE.

35. Ones, Twos, and Threes.

÷ Contains, or Divided by.

: Ratio of.....to.....

$$4 \div 2 = 2.$$

Read, "4 *contains* 2 2 times," or, "4 *divided by* 2 equals 2."

NOTE.—As we proceed, the pupil should become familiar with both forms of expression.

$$4 : 2 = 2.$$

Read, "The *ratio of* 4 *to* 2 equals 2."

6 apples = 6 × 1 apple.
6 apples = 3 × 2 apples.
6 apples = 2 × 3 apples.

Then, 6 contains how many 1's? $6 \div 1 = (\ )$? $6 : 1 = (\ )$?
6 contains how many 2's? $6 \div 2 = (\ )$? $6 : 2 = (\ )$?
6 contains how many 3's? $6 \div 3 = (\ )$? $6 : 3 = (\ )$?

A **Quotient** or **Ratio** tells how many times one number contains another.

NOTE.—No difference need be made between the words "*Quotient*" and "*Ratio*" at this time.

$0 \div 1 = 0$	$0 \div 2 = 0$	$0 \div 3 = 0$
$1 \div 1 = 1$	$2 \div 2 = 1$	$3 \div 3 = 1$
$2 \div 1 = 2$	$4 \div 2 = 2$	$6 \div 3 = 2$
$3 \div 1 = 3$	$6 \div 2 = 3$	$9 \div 3 = 3$
$4 \div 1 = 4$	$8 \div 2 = 4$	$12 \div 3 = 4$
$5 \div 1 = 5$	$10 \div 2 = 5$	$15 \div 3 = 5$
$6 \div 1 = 6$	$12 \div 2 = 6$	$18 \div 3 = 6$
$7 \div 1 = 7$	$14 \div 2 = 7$	$21 \div 3 = 7$
$8 \div 1 = 8$	$16 \div 2 = 8$	$24 \div 3 = 8$
$9 \div 1 = 9$	$18 \div 2 = 9$	$27 \div 3 = 9$

EXERCISE XXXV.

Copy and fill blanks:

1.	*2.*	*3.*
0 : 1 = ( )?	2 : 2 = ( )?	6 : 2 = ( )?
3 : 1 = ( )?	4 : 1 = ( )?	10 : 2 = ( )?
5 : 1 = ( )?	6 : 3 = ( )?	12 : 3 = ( )?
2 : 1 = ( )?	8 : 2 = ( )?	9 : 3 = ( )?
7 : 1 = ( )?	9 : 1 = ( )?	14 : 2 = ( )?

4.	*5.*	*6.*
8 : 1 = ( )?	18 : 2 = ( )?	27 : 3 = ( )?
15 : 3 = ( )?	0 : 3 = ( )?	3 : 3 = ( )?
6 : 1 = ( )?	0 : 2 = ( )?	1 : 1 = ( )?
16 : 2 = ( )?	12 : 2 = ( )?	21 : 3 = ( )?
18 : 3 = ( )?	24 : 3 = ( )?	4 : 2 = ( )?

Call results quickly:

7.	*8.*	*9.*	*10.*
6÷3	14÷2	5÷1	9÷1
8÷2	18÷3	10÷2	0÷3
9÷3	12÷3	16÷2	27÷3
12÷2	15÷3	24÷3	4÷2
6÷2	18÷2	21÷3	0÷2

NOTE.—In recitation, the teacher may say, "Divide 6 by 3; 8 by 2; 9 by 3," and so on, requiring the pupil to respond promptly each time with the result alone.

11.	*12.*	*13.*	*14.*
6:2	12:2	15:3	7:1
9:3	18:3	0:2	0:3
8:2	9:1	24:3	10:2
6:3	18:2	16:2	4:2
14:2	12:3	21:3	27:3

NOTE.—Teacher may say, "The ratio of 6 to 2; 9 to 3; 8 to 2," and so on.

Copy and fill blanks:

15.	*16.*	*17.*
6:( )=3?	12:( )=4?	15:( )=5?
8:( )=4?	9:( )=9?	21:( )=7?
9:( )=3?	6:( )=2?	5:( )=5?
12:( )=6?	18:( )=9?	24:( )=8?
18:( )=6?	14:( )=7?	27:( )=9?

To find results in 15, take $\frac{1}{3}$ of 6; $\frac{1}{4}$ of 8; $\frac{1}{3}$ of 9; $\frac{1}{6}$ of 12; $\frac{1}{6}$ of 18.

18.	*19.*	*20.*
( )÷3=9?	( )÷3=8?	( )÷2=9?
( )÷2=8?	( )÷2=6?	( )÷3=7?
( )÷2=5?	( )÷3=1?	( )÷2=2?
( )÷3=0?	( )÷2=7?	( )÷2=3?
( )÷3=5?	( )÷3=6?	( )÷1=7?

To find results in 18, take 9×3; 8×2; 5×2; 0×3; 5×3.

Mixed drill:

21. $6\times8-8\div2+\frac{1}{5}$ of $25-12\div3=(\ \ )$?
22. $8\times3+15\div3-\frac{1}{7}$ of $21+21\div3=(\ \ )$?
23. $27\div3+24\div3-\frac{3}{4}$ of $8-9\div3=(\ \ )$?
24. $5\times9+\frac{3}{7}$ of $14-18\div3+12\div2=(\ \ )$?
25. $7\times5-15\div3+24\div3-\frac{3}{5}$ of $15=(\ \ )$?
26. $8\times8-27\div3+16\div2+10\div2=(\ \ )$?

NOTE.—Perform these operations just as we have in the "Mixed drills" heretofore.

36. Problems.

EXAMPLES.

1. How many boxes will I need to box 12 books, if I can put only 3 books in 1 box?

Oral Solution: I will need as many boxes as 3 books are contained in 12 books, or 4 boxes.

Written Solution: (1) 12 books ÷ 3 books = ( )? (Question.)
(2) 12 books ÷ 3 books = 4.
∴ I will need 4 boxes.

NOTE.—The sign, ∴, is read "*Therefore.*"

2. John has \$3 and James has \$15. James has how many times as much money as John?

Oral Solution: The ratio of \$15 to \$3 is 5. Therefore James has 5 times as much money as John.

Written Solution: (1) \$15 : \$3 = ( )? (Question.)
(2) \$15 : \$3 = 5.
∴ James has 5 times as much as John.

NOTE TO TEACHER.—Either set of solutions may be used for either of the above problems, and for any of the problems in the following Exercise. Yet, a critical examination of the above examples will reveal the fact that in the first, *one* number is to be *divided* into parts—*a division problem;* and in the second, *two* numbers are to be compared as to size—*a ratio problem.* In the following Exercises the problems best suited for the *ratio solutions* will be marked, but the teacher is expected to exercise his own judgment and preference in the choice of solutions to be used.

3. Jane spends 10¢ for oranges, at 2¢ each. How many oranges does she buy?

Oral Solution: She buys as many oranges as 2¢ is contained times in 10¢, or 5 oranges.

Written Solution: (1) $10¢ \div 2¢ = (\ )$? (Question.)
(2) $10¢ \div 2¢ = 5$.
∴ she buys 5 oranges.

EXERCISE XXXVI.

Give both oral and written solutions:

1. How many trips must a man make to set 21 men across a river in a boat, if he can take 3 of them at each trip?

2. A woman cuts 24 yards of cloth into pieces of 3 yards each. How many pieces are there?

3. A man ate 3 eggs at a meal, until he had eaten 18 eggs. How many meals did it take?

4. A man put 12 birds into cages, putting 2 birds in each cage. How many cages were used?

5. Lucy is 3 years old and Nell is 9 years old. Nell is how many times as old as Lucy? (Ratio.)

6. A pole is 12 feet long, but its shadow is only 3 feet long. The pole is how many times as long as the shadow? (Ratio.)

7. A box that holds 27 bushels of wheat is how many times as large as one that holds 3 bushels of wheat? (Ratio.)

8. James has 14 rabbits, and May has 2 rabbits. James has how many times as many rabbits as May? (Ratio.)

9. If pencils are 3¢ each, how many can I buy for 21¢?

10. If apples are 2¢ each, how many can I buy for 16¢?

37. Fours, Fives, and Sixes.

12 apples = 3 × 4 apples.
10 apples = 2 × 5 apples.
18 apples = 3 × 6 apples.

Then, 12 contains how many 4's? $12 \div 4 = (\ )$? $12 : 4 = (\ )$?
10 contains how many 5's? $10 \div 5 = (\ )$? $10 : 5 = (\ )$?
18 contains how many 6's? $18 \div 6 = (\ )$? $18 : 6 = (\ )$?

$0 \div 4 = 0$	$0 \div 5 = 0$	$0 \div 6 = 0$
$4 \div 4 = 1$	$5 \div 5 = 1$	$6 \div 6 = 1$
$8 \div 4 = 2$	$10 \div 5 = 2$	$12 \div 6 = 2$
$12 \div 4 = 3$	$15 \div 5 = 3$	$18 \div 6 = 3$
$16 \div 4 = 4$	$20 \div 5 = 4$	$24 \div 6 = 4$
$20 \div 4 = 5$	$25 \div 5 = 5$	$30 \div 6 = 5$
$24 \div 4 = 6$	$30 \div 5 = 6$	$36 \div 6 = 6$
$28 \div 4 = 7$	$35 \div 5 = 7$	$42 \div 6 = 7$
$32 \div 4 = 8$	$40 \div 5 = 8$	$48 \div 6 = 8$
$36 \div 4 = 9$	$45 \div 5 = 9$	$54 \div 6 = 9$

EXERCISE XXXVII.

Copy and fill blanks:

1.	*2.*	*3.*
$20 : 4 = (\ )?$	$8 : 4 = (\ )?$	$30 : 5 = (\ )?$
$24 : 4 = (\ )?$	$4 : 4 = (\ )?$	$15 : 5 = (\ )?$
$16 : 4 = (\ )?$	$28 : 4 = (\ )?$	$40 : 5 = (\ )?$
$32 : 4 = (\ )?$	$0 : 4 = (\ )?$	$25 : 5 = (\ )?$
$12 : 4 = (\ )?$	$36 : 4 = (\ )?$	$0 : 5 = (\ )?$

4.	*5.*	*6.*
$5 : 5 = (\ )?$	$0 : 6 = (\ )?$	$30 : 6 = (\ )?$
$20 : 5 = (\ )?$	$36 : 6 = (\ )?$	$6 : 6 = (\ )?$
$35 : 5 = (\ )?$	$12 : 6 = (\ )?$	$42 : 6 = (\ )?$
$10 : 5 = (\ )?$	$48 : 6 = (\ )?$	$18 : 6 = (\ )?$
$45 : 5 = (\ )?$	$24 : 6 = (\ )?$	$54 : 6 = (\ )?$

7.	*8.*	*9.*
$12 \div (\ ) = 2?$	$10 \div (\ ) = 2?$	$42 \div (\ ) = 7?$
$20 \div (\ ) = 4?$	$16 \div (\ ) = 4?$	$40 \div (\ ) = 8?$
$28 \div (\ ) = 7?$	$25 \div (\ ) = 5?$	$36 \div (\ ) = 9?$
$15 \div (\ ) = 3?$	$24 \div (\ ) = 4?$	$45 \div (\ ) = 9?$
$8 \div (\ ) = 2?$	$32 \div (\ ) = 8?$	$54 \div (\ ) = 9?$

10.	*11.*	*12.*
( ) : 6 = 0 ?	( ) : 5 = 0 ?	( ) : 5 = 5 ?
( ) : 5 = 1 ?	( ) : 6 = 5 ?	( ) : 4 = 5 ?
( ) : 4 = 0 ?	( ) : 5 = 6 ?	( ) : 6 = 1 ?
( ) : 6 = 8 ?	( ) : 4 = 3 ?	( ) : 4 = 5 ?
( ) : 4 = 1 ?	( ) : 6 = 3 ?	( ) : 5 = 9 ?

Call results quickly:

13.	*14.*	*15.*	*16.*
12 ÷ 6	12 ÷ 4	40 ÷ 5	24 ÷ 6
10 ÷ 5	18 ÷ 6	28 ÷ 4	45 ÷ 5
42 ÷ 6	30 ÷ 6	25 ÷ 5	8 ÷ 4
20 ÷ 5	0 ÷ 6	36 ÷ 4	32 ÷ 4
16 ÷ 4	48 ÷ 6	15 ÷ 5	54 ÷ 6

17.	*18.*	*19.*	*20.*
0 : 6	36 : 6	20 : 4	12 : 4
0 : 5	16 : 4	0 : 4	24 : 4
25 : 5	42 : 6	30 : 5	4 : 4
5 : 5	32 : 4	6 : 6	18 : 6
30 : 6	40 : 5	48 : 6	45 : 5

Mixed drill:

21. $8 \times 9 - 30 \div 6 + \frac{1}{5}$ of $15 - 36 \div 4 = (\quad)$?

22. $\frac{1}{7}$ of $49 + 48 \div 6 - 54 \div 6 + 45 \div 5 = (\quad)$?

23. $8 \times 3 - 0 \div 6 + 25 \div 5 - 40 \div 5 = (\quad)$?

24. $25 - 35 \div 5 + 32 \div 4 - \frac{1}{9}$ of $36 = (\quad)$?

25. $72 - 18 \div 6 - 45 \div 5 - 30 \div 5 - 7 = (\quad)$?

26. $37 - 28 \div 4 - 20 \div 5 - 20 \div 4 - 18 \div 2 = (\quad)$?

Give oral or written solutions:

27. How many machines worth \$6 each can be bought for \$42?

28. I spent 45¢ for oranges, at 5¢ each. How many did I buy?

29. A 25¢.piece is worth how many times as much as a 5¢ piece? (Ratio.)

Question: A 25¢ piece is called what? A 5¢ piece is called what?

30. A man cuts 5 loads of wood in 1 day. How many days will he be in cutting 35 loads?

31. A girl can pick 6 gallons of berries in a day. In how many days can she pick 36 gallons?

32. A horse can walk 5 miles in an hour. In what time can he walk 40 miles?

33. Dora is 6 years old, and her father is 42 years old. The father is how many times as old as the child? (Ratio.)

34. A lady churns 6 pounds of butter each week. How long will she be in making 48 pounds of butter?

35. A pole 54 feet long is how many times as long as a pole 6 feet long? (Ratio.)

38. Sevens, Eights, and Nines.

21 apples = 3×7 apples.
24 apples = 3×8 apples.
27 apples = 3×9 apples.

Then, 21 contains 7 how many times? 21÷7=( )? 21:7=( )?
24 contains 8 how many times? 24÷8=( )? 24:8=( )?
27 contains 9 how many times? 27÷9=( )? 27:9=( )?

0÷7=0	0÷8=0	0÷9=0
7÷7=1	8÷8=1	9÷9=1
14÷7=2	16÷8=2	18÷9=2
21÷7=3	24÷8=3	27÷9=3
28÷7=4	32÷8=4	36÷9=4
35÷7=5	40÷8=5	45÷9=5
42÷7=6	48÷8=6	54÷9=6
49÷7=7	56÷8=7	63÷9=7
56÷7=8	64÷8=8	72÷9=8
63÷7=9	72÷8=9	81÷9=9

EXERCISE XXXVIII.

Copy and fill blanks:

1.	*2.*	*3.*
56 : 7 = ( )?	35 : 7 = ( )?	0 : 8 = ( )?
7 : 7 = ( )?	21 : 7 = ( )?	8 : 8 = ( )?
42 : 7 = ( )?	35 : 7 = ( )?	72 : 8 = ( )?
14 : 7 = ( )?	28 : 7 = ( )?	64 : 8 = ( )?
0 : 7 = ( )?	63 : 7 = ( )?	32 : 8 = ( )?

4.	*5.*	*6.*
24 : 8 = ( )?	0 : 9 = ( )?	72 : 9 = ( )?
40 : 8 = ( )?	9 : 9 = ( )?	18 : 9 = ( )?
16 : 8 = ( )?	45 : 9 = ( )?	81 : 9 = ( )?
32 : 8 = ( )?	27 : 9 = ( )?	63 : 9 = ( )?
56 : 8 = ( )?	54 : 9 = ( )?	36 : 9 = ( )?

7.	*8.*	*9.*
42 ÷ ( ) = 6?	56 ÷ ( ) = 8?	81 ÷ ( ) = 9?
14 ÷ ( ) = 2?	54 ÷ ( ) = 6?	64 ÷ ( ) = 8?
24 ÷ ( ) = 3?	56 ÷ ( ) = 7?	49 ÷ ( ) = 7?
36 ÷ ( ) = 4?	21 ÷ ( ) = 3?	72 ÷ ( ) = 9?
48 ÷ ( ) = 6?	32 ÷ ( ) = 4?	63 ÷ ( ) = 7?

10.	*11.*	*12.*
( ) : 7 = 1?	( ) : 9 = 0?	( ) : 9 = 3?
( ) : 7 = 0?	( ) : 8 = 1?	( ) : 8 = 0?
( ) : 7 = 4?	( ) : 7 = 6?	( ) : 8 = 2?
( ) : 8 = 5?	( ) : 9 = 2?	( ) : 9 = 1?
( ) : 9 = 3?	( ) : 7 = 3?	( ) : 7 = 5?

Call results quickly:

13.	*14.*	*15.*	*16.*
7 ÷ 7	0 ÷ 8	9 ÷ 9	56 ÷ 7
14 ÷ 7	16 ÷ 8	24 ÷ 8	72 ÷ 9
27 ÷ 9	63 ÷ 7	0 ÷ 9	56 ÷ 8
64 ÷ 8	32 ÷ 8	42 ÷ 7	81 ÷ 9
21 ÷ 7	18 ÷ 9	40 ÷ 8	48 ÷ 8

17.	*18.*	*19.*	*20.*
18 : 9	63 : 7	42 : 7	49 : 7
14 : 7	56 : 8	81 : 9	40 : 8
36 : 9	32 : 4	63 : 9	28 : 7
12 : 6	21 : 7	64 : 8	72 : 9
24 : 8	16 : 8	56 : 7	35 : 7

Mixed drill:

21. $59-56\div7-36\div9-14\div7-24\div8=(\quad)$?

22. $6\times9-54\div9-54\div6+21\div7+81\div9=(\quad)$?

23. $8-36\div9+72\div8+56\div8+42\div7=(\quad)$?

24. $7+72\div9-40\div8+54\div6-16\div8=(\quad)$?

25. $\frac{1}{8}$ of $72+49\div7+35\div7-0\div9+27\div9=(\quad)$?

26. $64\div8+81\div9-56\div7+14\div7=(\quad)$?

Give oral or written solutions:

27. 72 men are how many times 8 men? (Ratio.)

28. How many times can a merchant sell 9 chickens out of a flock of 54 chickens?

29. How many $5 bills will pay a debt of $35? (Ratio.)

30. In 1 peck there are 8 quarts. How many pecks in 40 quarts?

31. How many plows at $7 each can I buy for $35?

32. Reduce 56 quarts to pecks. (See 30 above.)

33. How many calves at $8 each can be bought for $72?

34. How many hogs at $7 each can be bought for $56?

PART II.

I. NOTATION AND NUMERATION.

39. Numbers of Three Figures.

99+1=**100**, One Hundred.

The third place from the right is for *hundreds*, and is called **Hundreds' Place.**

200, two hundred.
300, three hundred.
400, four hundred.
500, five hundred.
600, six hundred.
700, seven hundred.
800, eight hundred.
900, nine hundred.
325, three hundred twenty-five.
420, four hundred twenty.
708, seven hundred eight.
999, nine hundred ninety-nine.

NOTE.—Do not use "*and*" in reading these numbers.

1 hundred=10 tens.
1 ten=10 units.

A figure in hundreds' place represents **10 times** as many as the same figure in tens' place; a figure in tens' place represents **10 times** as many as the same figure in units' place.

NOTE.—If necessary, the teacher should use bundles of sticks or other objects to make this clear to the pupil.

EXERCISE XXXIX.

Read:

1.	*2.*	*3.*	*4.*	*5.*
124	259	333	411	521
137	234	307	401	508
186	222	398	433	503
139	240	327	444	516
195	209	386	409	548
188	265	305	484	539
107	214	370	438	594
111	255	315	476	580
165	203	375	418	565
117	212	300	422	519

6.	*7.*	*8.*	*9.*	*10.*
677	775	911	922	903
644	666	973	833	672
606	780	935	971	403
707	800	940	781	566
725	899	839	873	300
686	798	863	992	700
699	692	925	807	650
671	712	899	909	807
742	813	950	867	249
734	805	999	998	117

Write:

11.	*12.*
Two hundred forty.	Six hundred fifty-four.
Three hundred twenty-five.	Four hundred seventy-seven
Four hundred thirty-nine.	Eight hundred thirty-seven.
One hundred fifteen.	Three hundred ninety-eight
Two hundred eight.	Seven hundred twelve.
Three hundred one.	Seven hundred four.
Five hundred seventy-two.	Nine hundred twenty-three.
Four hundred eleven.	Five hundred sixty-five.
Nine hundred.	Eight hundred eighteen.
Six hundred eighty.	One hundred eleven.

13.	*14.*
Four hundred ninety-four.	Eight hundred seventy-two.
Two hundred nineteen.	Three hundred thirty-three.
Five hundred fifty-five.	One hundred fourteen.
Seven hundred seventy-nine.	Nine hundred three.
Eight hundred six.	Eight hundred seventy-six.
Nine hundred sixty-nine.	Six hundred twenty-one.
Eight hundred sixty.	Five hundred nineteen.
Seven hundred ten.	Seven hundred ninety-four.
Six hundred sixty-six.	Two hundred forty-eight.
Four hundred.	Nine hundred ninety-nine.

40. Periods.—Numbers larger than those we have yet used are written with more than three figures. As,

764256342.

Let us begin at the right side and separate this number into groups, putting three figures in each group. Thus,

764,256,342.

These groups are called **Periods.** The first period, or the one on the right, is **Units' Period;** the second period is **Thousands' Period;** the third period is **Millions' Period.** The last period, or the one on the left, sometimes has only one or two figures. Other periods must be full.

Millions.	*Thousands.*	*Units.*
764,	256,	342.

How many units? *Ans.*, 342.

How many thousand? *Ans.*, 256.

How many million? *Ans.*, 764.

Then, read the number. 764 million, 256 thousand, 342 (units).

NOTE.—Usually the name "*units*" is dropped; and in reading numbers we say "*thousand*," not thousands, and "*million*," not millions. But in speaking of the names of periods, we say "*thousands' period*," and "*millions' period.*"

EXAMPLES.

1. Read 204362.

Process: 204,362; read, 204 thousand, 362.

NOTE.—Before reading numbers always separate the figures into periods. *Now*, you should do this as above with commas; but *soon* you will be able to do this *mentally*, and without using commas.

2. Read 2000501.

Process: 2,000,501; read, 2 million, 501.
How many thousand?

3. Read 301002010.

Process: 301,002,010; 301 million, 2 thousand, 10.
In 002, how many *hundred?* how many *tens?* how many *units?* Does 002 thousand = 2 thousand?

4. Read 421060000.

Process: 421,060,000; 421 million, 60 thousand.

5. 342221; read, 342 thousand, 221.
6. 3425200; read, 3 million, 425 thousand, 200.
7. 42030006; read, 42 million, 30 thousand, 6.
8. 249000021; read, 249 million, 21.

EXERCISE XL.

First separate into periods with commas, and read; afterwards read from book:

1.	24240.	11.	1804309.
2.	5346.	12.	3407539.
3.	40224.	13.	47836820.
4.	248990.	14.	709846324.
5.	907684.	15.	673927432.
6.	777777.	16.	100100100.
7.	908809.	17.	190190019.
8.	111111.	18.	898989893.
9.	800008.	19.	112211011.
10.	760400.	20.	3003003.

Write:

21. Two thousand, three hundred thirty-four.

22. Thirty-seven thousand, five hundred twenty-six.

23. Nine hundred twelve thousand, eight hundred one.

24. Six hundred ninety-five thousand, one hundred forty-two.

25. Three million, two hundred seventy-eight thousand, four hundred eighty-nine.

26. Fifty million, seventy-seven thousand, six.

27. Two hundred million, one thousand, four hundred ninety-nine.

28. One hundred million, one hundred thousand, one.

29. Nine hundred sixty-nine million, two hundred twenty-six thousand, one hundred seventy-five.

30. One hundred three million, one hundred three thousand.

31. Two hundred nine million, twenty-nine.

32. Nine hundred eighty-five million, two hundred thousand, ten.

41. Expressing Dollars and Cents.—The *dollar-mark*, **$**, is always placed before the number with which it is used. Thus,

$1, one dollar;
$14, fourteen dollars;
$75, seventy-five dollars;
$840, eight hundred forty dollars.

Cents may be expressed with dollars, or with a *dollar-mark*. When so expressed, *two places* are always used for cents.

100 cents=1 dollar; then,

any number of cents less than a dollar may be written with two figures, of which the right-hand figure is for the units of cents, and the left-hand figure is for the tens of cents. A

period (.) is always placed between dollars and cents, and the word "*and*" is used there in reading. Thus,

\$2.10, 2 dollars and 10 cents;
\$3.07, 3 dollars and 7 cents;
\$.75, 75 cents;
\$.03, 3 cents;
\$347.50, 347 dollars and 50 cents.

Cents may also be written by using the *cent-mark*, ¢. Thus,

5¢, five cents;
17¢, seventeen cents;
85¢, eighty-five cents.

EXERCISE XLI.

Read:

1.	*2.*	*3.*	*4.*
\$5	\$3.20	\$.40	30¢
\$47	\$4.46	\$.84	5¢
\$224	\$17.04	\$.03	21¢
\$946	\$244.13	\$.50	70¢
\$3420	\$101.10	\$.99	99¢

Write with "\$":

5. Two dollars and forty cents; fifteen dollars and seventy-five cents; eight hundred fifteen dollars and thirty-four cents; twenty-five dollars and five cents; two cents; forty cents.

Write with "¢":

6. Thirty-four cents; twenty cents; nine cents; seventeen cents; ninety-five cents.

II. ADDITION.

42. Definitions.—Addition is the process of uniting two or more numbers into one. The numbers to be added

are called **Addends.** The result is called the **Sum** or **Amount.**

43. Uniting Addends Smaller than Ten.

EXERCISE XLII.

Copy and add:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*
5	8	8	4	9	5	8	2
7	3	6	9	3	8	7	7

9.	*10.*	*11.*	*12.*	*13.*	*14.*	*15.*	*16.*
4	7	8	9	5	6	3	1
5	3	3	2	7	8	7	9
6	5	5	4	9	3	2	8

17.	*18.*	*19.*	*20.*	*21.*	*22.*	*23.*	*24.*
8	2	9	3	3	4	9	5
7	3	5	4	8	5	5	4
6	7	4	7	7	8	9	9

25.	*26.*	*27.*	*28.*	*29.*	*30.*	*31.*	*32.*
4	5	8	3	9	2	8	9
7	5	0	4	2	3	9	8
5	4	6	6	7	5	7	6
6	4	5	8	8	9	6	1

Oral work:

NOTE.—In recitation, as the teacher calls the successive addends, the pupil should respond with successive sums. The drill should be continued till the response from the pupil is quick and accurate.

33. Add 3, 4, 5.
34. Add 8, 7, 3.
35. Add 9, 4, 7.
36. Add 3, 6, 8.
37. Add 5, 7, 3.
38. Add 8, 4, 2.
39. Add 5, 7, 9.
40. Add 1, 4, 7.

41. Add 3, 5, 6, 7, 8, 4.
42. Add 2, 8, 9, 3, 6, 4.
43. Add 5, 2, 6, 7, 9, 4.
44. Add 4, 3, 5, 8, 2, 6.
45. Add 6, 7, 5, 3, 9, 4.
46. Add 8, 1, 7, 6, 2, 9.
47. Add 3, 5, 4, 8, 6, 3.
48. Add 8, 7, 6, 7, 5, 8.

Add orally both ways:

	59.	*60.*	*61.*	*62.*	*63.*	*64.*	*65.*	*66.*	*67.*	*68.*
49.	3	5	4	7	1	9	6	8	2	7
50.	4	6	1	0	9	5	8	2	7	4
51.	6	9	8	9	2	3	1	4	6	5
52.	2	2	6	2	7	9	4	8	5	0
53.	9	4	9	3	5	8	6	3	7	2
54.	7	8	2	6	4	5	0	9	3	8
55.	8	1	5	4	6	7	2	6	3	5
56.	5	5	7	3	8	1	9	4	6	9
57.	1	7	3	5	6	8	4	9	7	9
58.	3	1	2	9	4	5	6	7	9	8

Give oral solutions:

69. John paid 9¢ for a slate, 6¢ for a pencil, 5¢ for a sponge, and 8¢ for pens. How much did he spend in all?

Oral Solution: He spent in all the sum of 9¢, 6¢, 5¢, and 8¢, or 28¢.

(The pupil should be permitted to pause after pronouncing the word "*or*," to find the sum.)

70. I bought 4 books, paying for the first $7, for the second $3, for the third $5, and for the fourth $6. How much did I pay for all?

71. A man has 7 apple trees, 5 pear trees, 4 plum trees, and 9 cherry trees. How many trees in all?

72. Charles is 5 years old, Nell is 4 years older than Charles,

James is 6 years older than Nell, and Philip is 8 years older than James. How old is Philip?

73. A boy reads 9 pages on Monday, 4 pages on Tuesday, 7 pages on Wednesday, 6 pages on Thursday, 8 pages on Friday, and 9 pages on Saturday. How many pages has he read?

74. There are 8 books on the first shelf, 9 books on the second, 7 books on the third, and 8 books on the fourth. How many books in all?

75. How much would I have to pay for a 5-cent ball, a 4-cent orange, an 8-cent pencil, and a 7-cent tablet?

76. There are 8 pupils in one class, 4 pupils in another, 7 pupils in another, and 5 pupils in another. How many pupils in all?

44. Uniting Addends Larger than Nine.— Write the second number under the first, the third under the second, and so on. Place units under units, tens under tens, and so on. Draw a line below. Add the columns separately, beginning with units. It is usually more convenient to add upwards.

EXAMPLES.

1.

43 }	...Addends.
25 }	
68	..... Sum.

Explanation complete: (1) 5 units + 3 units = 8 units. Write the *8* below.
(2) 2 tens + 4 tens = 6 tens. Write the *6* below.

In this example, the sum obtained by adding each column is smaller than 10, and is written below with one figure. When the sum obtained by adding any column is larger than 9, then it cannot be written below with one figure. We have now to learn the process called "carrying." We know that,

10 units = 1 ten,

10 tens = 1 hundred, etc.

That is, *10 of any certain place make 1 of the next higher place.*

"*Carrying*" consists in taking the 10's from one place and adding them as 1's to the next higher place.

2.
46
27
73

Explanation complete: (1) 7 units + 6 units = 13 units, or 1 ten and 3 units. Write the *3* below, and carry the 1 ten to the next column.

(2) 1 ten (carried) + 2 tens + 4 tens = 7 tens. Write the 7 below.

3.
98
77
48
223

Explanation shortened: (1) 8 + 7 + 8 = 23. Write the *3* below and carry 2.

(2) 2 (carried) + 4 + 7 + 9 = 22. Write the *22* below. Always write the whole sum obtained by adding the last column.

4.
$8.37
$17.93
$26.30

Explanation: (1) 3 units + 7 units = 10 units, or 1 ten and 0 units. Write the *0* below, and carry the 1 ten.

(2) 1 ten (carried) + 9 tens + 3 tens = 13 tens, or 1 hundred and 3 tens. Write the *3* below and carry the 1 hundred (1 hundred cents = $1).

(3) 1 (carried) + 7 + 8 = 16. Write the *6* below, and carry the 1.

(4) 1 (carried) + 1 = 2. Write the *2* below.

5.	*6.*	*7.*	*8.*	*9.*
79	85	78	79	546
64	47	49	83	607
143	132	127	162	1153

NOTE.—Teacher, explain all these examples carefully before taking up the Exercise.

EXERCISE XLIII.

Copy and add:

1.	*2.*	*3.*	*4.*	*5.*	*6.*
67	44	98	72	51	87
59	88	79	64	99	77

7.	*8.*	*9.*	*10.*	*11.*	*12.*
84	48	55	75	66	97
39	56	73	94	86	45

13.	*14.*	*15.*	*16.*	*17.*	*18.*
28	46	97	43	55	63
79	84	86	88	95	29

19.	*20.*	*21.*	*22.*	*23.*	*24.*
49	34	56	86	29	35
66	77	87	99	95	68

25.	*26.*	*27.*	*28.*	*29.*	*30.*
26	43	72	38	79	88
59	57	65	99	34	35
84	66	39	56	86	74

31.	*32.*	*33.*	*34.*	*35.*	*36.*
46	78	19	36	78	49
59	83	72	94	87	50
34	27	44	75	29	36

37.	*38.*	*39.*	*40.*	*41.*	*42.*
55	95	70	84	97	28
61	25	88	6	87	38
88	45	39	38	77	48

43.	*44.*	*45.*	*46.*	*47.*	*48.*
128	263	439	846	340	873
346	444	372	271	209	24
295	339	798	924	334	549

49.	*50.*	*51.*	*52.*	*53.*	*54.*
421	428	921	888	492	354
342	478	436	707	725	643
709	548	53	563	346	820

55.	*56.*	*57.*	*58.*	*59.*
147	3421	707	25	8604
584	590	5423	946	5227
1362	1842	36	4321	6666

60.	*61.*	*62.*	*63.*	*64.*
347	846	9063	7086	5555
1594	50	488	1944	6060
6004	976	9999	3221	7777
7321	1354	69	8888	1111

65.	*66.*	*67.*	*68.*	*69.*
$5.20	$12.45	$72.60	$66.60	$243.00
$7.42	$18.21	$34.20	$75.30	$83.24
$8.99	$24.97	$50.50	$94.25	$17.60
$12.90	$5.90	$39.80	$65.95	$80.95

Solve as on page 24:

70. In a certain school building there are four rooms: in the first room there are 29 pupils; in the second, 48 pupils; in the third, 70 pupils; and in the fourth, 69 pupils. How many pupils in all?

71. I paid for a buggy $75, for a horse $64.50, for a harness $27.40, for lap-robe and whip $3.25. How much did I pay altogether?

72. I owe Mr. Jones $246.50, Mr. Brown $121.75, Mr. Smith $98.10. How much money will pay all these debts?

73. In one flock there are 46 birds; in another, 78 birds; in another, 98 birds; and in another, 246 birds. How many birds in all?

74. One boy has 56¢; another, 84¢; another, 95¢; and another, $1.20. How much money have all?

NOTE.—In writing to add, express the numbers all with "$" instead of "¢."

75. Four men compare their money: A has $542; B has $274 more than A; C has $221 more than B; and D has $498 more than C. How much has B? C? D?

76. Mr. Taylor owns 3 farms: in the first there are 64 acres; in the second, 127 acres; in the third, 98 acres. How many acres does he own?

77. Mr. Badley deposits in the bank on Monday $483, on Tuesday $79, on Wednesday $349, on Thursday $873, on Friday $500, on Saturday $871. How much does he deposit in all?

III. SUBTRACTION.

45. Definitions.—Subtraction is the process of taking one number from another. The number taken is called the **Subtrahend.** The number from which the subtrahend is taken is called the **Minuend.** The result of subtraction is called the **Remainder** or **Difference.**

46. Subtracting, when the Subtrahend is Smaller than Ten.

EXERCISE XLIV.

Copy and subtract:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*
10	9	7	12	15	13	18	11
7	5	7	8	5	7	9	5

9.	*10.*	*11.*	*12.*	*13.*	*14.*	*15.*	*16.*
14	18	16	12	9	17	13	16
9	6	8	5	6	4	9	7

Call results quickly:

17. Take 6 from 8, 15, 14, 12, 18, 11, 9, 17.

18. Take 4 from 11, 9, 15, 13, 17, 14, 10, 12.

19. Take 7 from 16, 10, 12, 14, 19, 11, 15, 18.
20. Take 8 from 12, 15, 13, 9, 11, 14, 10, 17.
21. Take 5 from 7, 12, 9, 15, 13, 11, 17, 14.
22. Take 9 from 12, 18, 16, 13, 11, 17, 19, 15.

Give oral solutions:

23. Nell had 12 chickens, but sold 6 of them. How many had she left?

Oral Solution: She had left the difference between 12 chickens and 6 chickens, or 6 chickens.

24. James killed 8 birds out of a flock of 17 birds. How many flew away?

25. My class now has 7 pupils. How many more must join to make 16 pupils?

26. I bought a slate for 8¢, and threw down to the merchant 10¢. How much change did I get?

27. Clara is now 9 years old. In how many years will she be 16 years old?

28. James is 18 years old, and Charles is 7 years younger than James. How old is Charles?

29. George has 5¢. How much must his mother give him so that he may have 11¢?

30. A boy, when asked the distance to town, replied, "If it were 9 miles farther, it would be 18 miles." How far is it to town?

47. Subtracting, when the Subtrahend is Larger than Nine.—Write the subtrahend under the minuend. Place units under units, tens under tens, and so on. Draw a line below. Always begin with units to subtract.

EXAMPLES.

1. 76.....Minuend.
45.....Subtrahend.
31.....Remainder.

Explanation complete: (1) 6 units − 5 units = 1 unit. Write *1* below. (2) 7 tens − 4 tens = 3 tens. Write *3* below.

In this example each lower figure is smaller than the one above it, and all that is required is to take the difference and write it below. It often happens that a lower figure is larger than the one above it. We must then take ("borrow") from the next higher place in the minuend and add to the smaller figure. This is called "*borrowing.*" But it is not *borrowing;* we simply *take* it.

2. 43
26
17

Explanation complete: (1) We cannot take 6 units from 3 units; but we can take 1 ten of the 4 tens, and add it to the 3 units. This will make 13 units. Then, 13 units − 6 units = 7 units. Write the 7 below. (2) One ten has been taken from the 4 tens, which leaves but 3 tens in the minuend. 3 tens − 2 tens leaves 1 ten. Write the *1* below.

3. 704
375
329

Explanation complete: (1) We cannot take 5 units from 4 units, and there are no tens in the minuend; then we must take 1 hundred from the 7 hundred. 1 hundred = 10 tens. Now, we can take 1 of the 10 tens and add it to the 4 units. 1 ten + 4 units = 14 units. 14 units − 5 units = 9 units. Write the *9* below.

(2) Of the 10 tens we took from hundreds' place, we have used 1 ten and have 9 tens left. 9 tens − 7 tens = 2 tens. Write the *2* below.

(3) We have used 1 hundred of the 7 hundred; then we have only 6 hundred left. 6 hundred − 3 hundred = 3 hundred. Write the *3* below.

4. 8801
5674
3127

Explanation shortened: (1) Looking upon *1*, think 11 (why?). 11 − 4 = 7. Write the 7 below.

(2) Looking upon the 0, think 10, then 9 (why?). 9 − 7 = 2. Write the *2* below.

(3) Looking upon the 8, think 7 (why?). 7 − 6 = 1. Write the *1* below.

(4) 8 − 5 = 3. Write the *3* below.

NOTE.—If the pupil does not understand, the teacher must complete the explanation.

5. $8.35
$3.85
$4.50

Explanation: (1) 5 units − 5 units = 0. Write the *0* below.

(2) We cannot take 8 tens from 3 tens; then, we must take $1 of the $8. ($1 = 1 hundred cents.) 1 hundred + 3 tens = 13 tens. 13 tens − 8 tens = 5 tens. Write the *5* below.

(3) 7 − 3 = 4. Write the *4* below.

6.	*7.*	*8.*	*9.*	*10.*
83	71	246	304	$43.75
59	24	198	59	$12.95
24	47	48	245	$30.80

EXERCISE XLV.

Copy and subtract:

1.	*2.*	*3.*	*4.*	*5.*	*6.*
73	75	74	76	71	70
27	36	44	59	66	19

7.	*8.*	*9.*	*10.*	*11.*	*12.*
81	82	83	84	85	86
15	27	35	49	76	38

13.	*14.*	*15.*	*16.*	*17.*	*18.*
65	64	60	63	62	61
48	39	23	44	57	56

19.	*20.*	*21.*	*22.*	*23.*	*24.*
91	95	92	97	98	90
14	37	55	39	49	58

25.	*26.*	*27.*	*28.*	*29.*	*30.*
107	224	336	248	129	300
78	97	99	159	74	236

31.	*32.*	*33.*	*34.*	*35.*	*36.*
421	256	538	292	473	801
398	167	295	164	290	507

37.	*38.*	*39.*	*40.*	*41.*	*42.*
763	150	471	835	752	591
409	127	359	583	594	327

43.	*44.*	*45.*	*46.*	*47.*
1024	2504	3000	1473	8421
594	1999	2462	1287	5988

48.	*49.*	*50.*	*51.*	*52.*
$10.05	$15.64	$28.30	$79.62	$88.07
$9.88	$1.08	$15.75	$58.77	$10.69

53. From 8441 take 7989.

54. From 1846 take 1290.

55. From 1000 take 837.

56. From 1001 take 994.

57. From 1201 take 555.

58. From 7010 take 5063.

59. From 6240 take 2986.

60. From 4653 take 3278.

61. From 2962 take 2786.

62. From 1800 take 1053.

63. I bought a horse for $65.50, and sold him for $73.25. How much did I gain?

$73.25
$65.50
$7.75 *Ans.*

64. I bought a horse for $64, and sold him for $55.50. How much did I lose?

65. A is 73 years old, and B is 59 years old. What is the difference between their ages?

66. Charles says to James, "I have $82;" James says, "I lack $44.50 of having as much as you have." How much money has James?

67. What number added to 39 will make 121?

68. What number added to 63 will make 130?

69. I bought a farm, and sold it for $840. If I gained $175, what did the farm cost?

70. It cost me $734.50 to build my house, but my barn cost $460.75 less than the house. How much did the barn cost?

IV. MULTIPLICATION.

48. Definitions.—Multiplication is a process, shorter than addition, for finding the sum when one number is to be used as an addend several times. The number to be used as the addend is called the **Multiplicand.** The number showing how many times the multiplicand is to be used is called the **Multiplier.** The result of multiplication is called the **Product.** The expression, "*Multiplied by*," used to indicate multiplication, means that the number placed *before* it is to be used as the *multiplicand* and the number placed *after* it is to be used as the *multiplier*.

49. Multiplying when the Multiplier is Smaller than Ten.—*Begin with units to multiply.*

EXAMPLES.

1.

23.... Multiplicand.
2.....Multiplier.
46..... Product.

Explanation: (1) 2×3 units=6 units. Write the *6* below.
(2) 2×2 tens=4 tens. Write the *4* below.

2.

46
6
276

Explanation complete: (1) 6×6 units=36 units, or 3 tens and 6 units. Write the *6* below and carry the 3.
(2) 6×4 tens=24 tens. 24 tens+3 tens (carried)=27 tens. Write *27* below.

3.

$$\begin{array}{r} 349 \\ 8 \\ \hline 2792 \end{array}$$

Explanation shortened: (1) $8\times9=72$. Write the *2* below and carry the 7.

(2) $8\times4=32$. $32+7=39$. Write the *9* below and carry the 3.

(3) $8\times3=24$. $24+3=27$. Write the *27* below.

4.

$$\begin{array}{r} \$12.56 \\ 7 \\ \hline \$87.92 \end{array}$$

Explanation: 7×6 units $=42$ units, or 4 tens and 2 units. Write the *2* below and carry the 4.

(2) 7×5 tens $=35$ tens. 35 tens $+4$ tens $=39$ tens, or 3 hundred and 9 tens. Write the *9* below and carry the 3. (3 hundred cents $=\$3$.)

(3) $7\times2=14$. $14+3=17$. Write the *7* below and carry the 1.

(4) $7\times1=7$. $7+1=8$. Write the *8* below.

5.	*6.*	*7.*	*8.*	*9.*
38	76	243	476	\$34.60
5	6	8	7	9
190	456	1944	3332	\$311.40

NOTE.—Teacher, explain the examples thoroughly before taking up the Exercise.

EXERCISE XLVI.

Copy and multiply:

1.	*2.*	*3.*	*4.*	*5.*	*6.*
47	59	84	31	72	95
2	3	4	5	6	7
7.	*8.*	*9.*	*10.*	*11.*	*12.*
49	56	73	94	82	69
8	9	7	6	7	5
13.	*14.*	*15.*	*16.*	*17.*	*18.*
88	76	59	45	94	87
4	6	5	9	7	8

19.	*20.*	*21.*	*22.*	*23.*	*24.*
97	66	28	39	52	95
8	7	6	7	8	9

Find answers:

25. 72 multiplied by 9=( )?
26. 84 multiplied by 8=( )?
27. 90 multiplied by 7=( )?
28. 37 multiplied by 9=( )?
29. 69 multiplied by 6=( )?
30. 164 multiplied by 8=( )?
31. 249 multiplied by 6=( )?
32. 304 multiplied by 7=( )?
33. 5×346=( )?
34. 7×408=( )?
35. 8×591=( )?
36. 9×500=( )?
37. 9×384=( )?
38. 8×539=( )?
39. 7×284=( )?
40. 6×297=( )?

Solve as in Article 26:

41. What will 6 buggies cost, at $55 each?

42. What will 9 wagons cost, at $75 each?

43. What will 8 books cost, at $.87 each?

44. A man earns $85 a month for 9 months. How much does he make in all?

45. How far will a train run in 7 hours, running 36 miles per hour?

46. I bought 8 horses, at $65 per head. How much did they cost me?

47. A boy sold 8 melons, at 25¢ each. How much money did he get for them?

48. There are 6 shelves in a book case, and 28 books on each shelf. How many books in the case?

49. How much will a boy earn in 9 days, at 48¢ per day?

50. How many days in 6 years of 365 days each?

Length of 6 years = ( ) days? (Question.)

50. Multiplying by Multipliers Larger than Nine.—Multiply by each figure of the multiplier, beginning with units. *Always place the right-hand figure of each product under that figure of the multiplier used. Add these (partial) products for the complete product.*

EXAMPLES.

1.

47
24
188 } Partial products.
94 }
1128.. Complete product.

NOTE.—The "4" in the second partial product is placed directly below the "2" of the multiplier.

NOTE.—The "Long Way" given in the following examples is for explanation, and not to be practiced by the pupil.

Long Way.

2.

57
40
00
228
2280

NOTE.—In the *Short Way*, we place the *4* under the *7*, and the *0* to the right of the 4. Bring down the *0* and multiply by *4* only.

Short Way.

2.

57
40
2280

Long Way.

3.

370
500
000
000
1850
185000

NOTE.—In the *Short Way*, we place the *5* under the *7*, letting the 0's fall to the right. Multiply by the *5*, and bring down three 0's—one for the 0 in the multiplicand, and two for the 0's in the multiplier.

Short Way.

3.

370
500
185000

Long Way. *Short Way.*

4.

246	
502	
492	
000	
1230	
123492	

NOTE.—We need not multiply by the 0 in the multiplier. But we must be careful to put the *0* of the partial product obtained by multiplying by the *5* under the *5*.

4.

246
502
492
1230
123492

5.

246
100
24600

6.

420
2400
168
84
1008000

7.

273
30600
1638
819
8353800

EXERCISE XLVII.

Copy and multiply:

1.	*2.*	*3.*	*4.*	*5.*	*6.*
37	43	57	84	91	77
25	36	73	22	38	34
7.	*8.*	*9.*	*10.*	*11.*	*12.*
52	65	34	28	47	59
16	12	18	24	36	75
13.	*14.*	*15.*	*16.*	*17.*	*18.*
42	53	64	71	95	86
24	35	46	17	59	68

Find products:

19. 30×57*

20. 40×95

21. 20×72

22. 50×84

23. 60×93

24. 70×64

25. 40×80

26. 45×90

27. 30×107

28. 100×64

29. 300×306

30. 400×700

*Read "30 times 57;" 30 is the multiplier.

Find products:

31.	$18 \times \$12.20$	*37.*	$65 \times \$94.75$
32.	$24 \times \$8.08$	*38.*	$47 \times \$38.50$
33.	$46 \times \$10.09$	*39.*	$34 \times \$15.06$
34.	$32 \times \$64.50$	*40.*	$55 \times \$19.80$
35.	$40 \times \$17.60$	*41.*	$65 \times \$27.30$
36.	$27 \times \$2.08$	*42.*	$93 \times \$40.50$

After finding products, use as an oral drill:

43.	10×0	*53.*	11×0	*63.*	12×0
44.	10×1	*54.*	11×1	*64.*	12×1
45.	10×2	*55.*	11×2	*65.*	12×2
46.	10×3	*56.*	11×3	*66.*	12×3
47.	10×4	*57.*	11×4	*67.*	12×4
48.	10×5	*58.*	11×5	*68.*	12×5
49.	10×6	*59.*	11×6	*69.*	12×6
50.	10×7	*60.*	11×7	*70.*	12×7
51.	10×8	*61.*	11×8	*71.*	12×8
52.	10×9	*62.*	11×9	*72.*	12×9

NOTE.—Teacher, require the pupil to be thorough in the last 30 points. Have them learned as a table.

Copy and multiply, using the whole multiplier at one time:

73.	*74.*	*75.*	*76.*	*77.*	*78.*
59	56	54	52	55	58
12	11	12	11	11	12

79.	*80.*	*81.*	*82.*	*83.*	*84.*
48	47	57	65	76	49
12	12	11	12	11	11

85.	*86.*	*87.*	*88.*	*89.*	*90.*
127	246	521	430	570	820
11	12	12	12	11	11

91.	*92.*	*93.*	*94.*	*95.*
$12.65	$47.84	$59.63	$40.80	$75.30
12	12	11	11	12

Find products:

96. 346×753
97. 240×598
98. 407×867
99. 488×921
100. 344×240
101. 426×989

51. Continued Multiplication.—Often we multiply one number by another and their product by a third number, and so on. Thus:

$$4\times5=20$$
$$6\times20=120$$

Such operations may be indicated in one statement. Thus:

$$6\times4\times5=120$$

Read, 6 times 4 times 5 equals 120.

EXERCISE XLVIII.

Perform the operations indicated:

1. 3×4×5.
2. 6×7×12.
3. 9×13×17.
4. 12×10×200.
5. 18×24×36.
6. 16×43×91.
7. 12×74×321.
8. 8×212×342.
9. 9×99×999.
10. 24×6×700.
11. 18×349×250.
12. 246×374×521.
13. 7×25×36×73.
14. 8×18×28×38.
15. 92×91×90×89.
16. 121×242×346.

52. To Multiply an Equation.—When two numbers are equal and are written with the sign, =, between them, the expression is called an **equation**; the number on the left is the **first** or **left member**; that on the right, the **right** or **second member.** *Both members of an equation may be multiplied by the same number and the results will be equal.*

EXAMPLES.

1. 1 bushel = 4 pecks. Multiply by 5.

Process: (1) 1 bushel = 4 pecks.
5 × (1) = (2)* 5 bushels = 20 pecks.

NOTE.—The teacher should take pains to see that the pupil understands just what has been done to equation (1) to obtain equation (2).

2. Weight of 1 load = 2500 pounds. Multiply by 12.

Process: (1) Weight of 1 load = 2500 pounds.
12 × (1) = (2) weight of 12 loads = 30000 pounds.

NOTE.—When numbers are small, the multiplying should be done mentally; when large, they may be set somewhere else for multiplying.

3. \$5 = cost of 2 books. Multiply by 6.

Process: (1) \$5 = cost of 2 books.
6 × (1) = (2) \$30 = cost of 12 books.

4. Price of 12 hats = \$30. Multiply by 7.

Process: (1) Price of 12 hats = \$30.
7 × (1) = (2) Price of 84 hats = \$210.

5. Cost of 1 yard = \$$\frac{1}{8}$. Multiply by 5.

Process: (1) Cost of 1 yard = \$$\frac{1}{8}$.
5 × (1) = (2) Cost of 5 yards = \$$\frac{5}{8}$.

EXERCISE XLIX.

1. 1 pound = 16 ounces. Multiply by 7.
2. 1 mile = 1760 yards. Multiply by 8.
3. 12 inches = 1 foot. Multiply by 9.
4. 2 pecks = 16 quarts. Multiply by 12.
5. 20 pints = 10 quarts. Multiply by 35.
6. Cost of 1 hat = \$2.50. Multiply by 16.
7. Cost of 10 melons = \$1. Multiply by 8.

*The expression, 5 × (1) = (2), should be read, five times *equation one* equals *equation two*.

8. $2.40 = cost of 6 buckets. Multiply by 11.

9. 2430 pounds = weight of 1 load. Multiply by 13.

10. $550 = price of 10 horses. Multiply by 14.

53. Problems.

EXAMPLES.

1. What will 75 buggies cost at $82.50 each?

Solution: (1) Cost of 75 buggies = $( )? (Question.)
(2) Cost of 1 buggy = $82.50. (Basis.)
75 × (2) = (3) Cost of 75 buggies = $6187.50. (Ans.)

Mechanical Work.

$82.50
75
4125
5775
$6187.50

NOTE.—When the teacher finds it necessary, the pupil may be required to accompany the solution with the "*Mechanical Work.*" But if the pupil has had the proper drill in quick, accurate multiplying, he need not preserve his "mechanical work."

2. In a certain field there are 268 rows of corn with 147 hills to the row. How many hills in the field?

Solution: (1) No. of hills in 268 rows = ( )? (Question.)
(2) No. of hills in 1 row = 147. (Basis.)
268 × (2) = (3) No. of hills in 268 rows = 39396. (Ans.)

3. How much oil will a well yield in one year (313) days at the rate of 1835 gallons per day?

Solution: (1) Yield for 313 days = ( ) gallons? (Question.)
(2) Yield for 1 day = 1835 gallons. (Basis.)
313 × (2) = (3) Yield for 313 days = 574355. (Ans.)

EXERCISE L.

1. Find the value of 190 horses, at $75 each.

2. Find the cost of 67 reapers, at $95 each.

3. Find the weight of 47 loads of wheat, of 2840 pounds each.

4. What will 325 tons of coal cost, at $3.75 per ton?

5. What will 28 plows cost, at $12.90 each?

6. What will 15 sewing-machines cost, at $24.50 each?

7. There are 1440 minutes (min.) in 1 day. How many minutes in 1 year of 365 days?

8. One hogshead contains 63 gal. How many gallons will 40 hogsheads contain?

9. In 1 bushel there are 64 pints. How many pints in 36 bushels?

10. A sewing-machine makes 125 stitches in 1 minute. At that rate, how many stitches will it make in 178 minutes?

11. A man hires to work 180 days, at $1.75 per day. How much money will he earn in that time?

12. There are 5280 feet in 1 mile. How many feet in 295 miles?

54. Table of Dry Measures.—These measures are used to measure quantities of grain, fruits, seeds, etc. *Commit the table to memory.*

TABLE.

1 bushel (bu.)=4 pecks (pk.)
1 pk.=8 quarts (qt.)
1 qt.=2 pints (pt.)

NOTE.—Pictures of cups, and other measures, are omitted in this book. The author believes that if the pupil is familiar with (say) a pint cup, the picture is useless; and if the pupil is not familiar with the real cup, a picture would be of little use,—as *size*, rather than *form*, is the real object of knowledge. The majority of pupils will be found to be familiar with these measures. When they are not, the teacher should use real measures and experiments, as far as possible, to convey a knowledge of the true size of the measure.

EXAMPLES.

NOTE.—Hereafter the "*Question*" will usually be omitted in the solution of examples, but the "*Basis*" will be marked throughout the book. The teacher must be the judge as to when the pupil can omit the "*Question*" from his solutions.

1. Reduce 3 bushels to pecks.

Solution: (1) 1 bu. = 4 pk. (Basis.)
3 × (1) = (2) 3 bu. = 12 pk. (Ans.)

NOTE.—We get the *Basis* from the table.

2. Reduce 5 pecks 4 quarts to pints.

NOTE.—Reduce the pecks to quarts, add the quarts, then reduce to pints.

Solution: (1) 1 pk. = 8 qt. (Basis.)
5 × (1) = (2) 5 pk. = 40 qt.
(3) 40 qt. + 4 qt. = 44 qt.
(4) 1 qt. = 2 pt. (Basis.)
44 × (4) = (5) 44 qt. = 88 pt. (Ans.)

3. What will 7 bushels 3 pecks of apples cost, at $.20 per peck?

Solution: (1) 1 bu. = 4 pk. (Basis.)
7 × (1) = (2) 7 bu. = 28 pk.
(3) 28 pk. + 3 pk. = 31 pk.
(4) Cost of 1 pk = $.20. (Basis.)
31 × (4) = (5) Cost of 31 pk. = $6.20. (Ans.)

4. Reduce 1 bushel 3 pecks 7 quarts 1 pint to pints.

Solution: (1) 1 bu. = 4 pk. (Basis.)
(2) 4 pk. + 3 pk. = 7 pk.
(3) 1 pk. = 8 qt. (Basis.)
7 × (3) = (4) 7 pk. = 56 qt.
(5) 56 qt. + 7 qt. = 63 qt.
(6) 1 qt. = 2 pt. (Basis.)
63 × (6) = (7) 63 qt. = 126 pt.
(8) 126 pt. + 1 pt. = 127 pt. (Ans.)

EXERCISE LI.

1. Reduce 16 quarts to pints.

2. Reduce 12 pecks to quarts.

3. Reduce 27 bushels to pecks.

4. Reduce 17 pecks 5 quarts to pints.

5. Reduce 29 bushels 2 pecks to quarts.

6. Reduce 84 bushels to pints.

7. Reduce 30 bushels 7 quarts to pints.

8. A boy sold 2 bushels of cherries at 10¢ per quart. How much did they bring?

9. I sold 12 bushels and 3 pecks of potatoes at 15¢ a peck. How much did I get for them?

10. Mr. Brown has 4 bushels 1 peck 3 quarts of plums, for which he asks 12¢ a quart. How much money will buy them?

55. Table of Liquid Measures.—These measures are used in measuring *Liquids*, like water and milk. *Commit the table to memory.*

TABLE.

1 gallon (gal.) = 4 quarts (qt.)
1 qt. = 2 pints (pt.)
1 pt. = 4 gills (gi.)

NOTE.—The *quart* and *pint* in this table are smaller than in Dry Measure table.

EXAMPLES.

1. Reduce 7 gallons to gills.

Solution: (1) 1 gal. = 4 qt. (Basis.)
7 × (1) = (2) 7 gal. = 28 qt.
(3) 1 qt. = 2 pt. (Basis.)
28 × (3) = (4) 28 qt. = 56 pt.
(5) 1 pt. = 4 gi. (Basis.)
56 × (5) = (6) 56 pt. = 224 gi. (Ans.)

2. What is the price of 12 gallons of milk at 5 cents a quart?

Solution: (1) 1 gal. = 4 qt. (Basis.)
12 × (1) = (2) 12 gal. = 48 qt.
(3) Price of 1 qt. = $.05. (Basis.)
48 × (3) = (4) Price of 48 qt. = $2.40. (Ans.)

EXERCISE LII.

1. Reduce 12 pints to gills.
2. Reduce 17 quarts to gills.
3. Reduce 24 gallons to quarts.
4. Reduce 18 gallons to pints.
5. Reduce 46 gallons 3 quarts to gills.
6. Reduce 244 quarts 3 gills to gills.
7. Reduce 8 gallons 3 quarts 1 pint to pints.
8. What will 5 gallons of syrup cost, at $.15 per quart?
9. What will 2 bushels 2 pecks of cherries sell for, at $.10 per quart? (Use Dry Measure Table. Why?)
10. What is 5 gallons of ice cream worth, at $.12 a pint? (Ice cream is measured as a liquid.)

56. Table of Avoirdupois Weights.—These weights are used in weighing all ordinary articles which are bought and sold by weight, such as groceries, meats, live stock, etc. *Commit the table to memory.*

TABLE.

1 ton (T.) = 20 hundredweight (cwt.).
1 cwt. = 100 pounds (lb.).
1 lb. = 16 ounces (oz.).

EXAMPLES.

1. Reduce 2 tons 4 hundredweight 44 pounds to ounces.

Solution: (1) 1 T. = 20 cwt. (Basis.)
2 × (1) = (2) 2 T. = 40 cwt.
(3) 40 cwt. + 4 cwt. = 44 cwt.

(4) 1 cwt. = 100 lb. (Basis.)
44 × (4) = (5) 44 cwt. = 4400 lb.
(6) 4400 lb. + 44 lb. = 4444 lb.
(7) 1 lb. = 16 oz. (Basis.)
4444 × (7) = (8) 4444 lb. = 71104 oz. (Ans.)

2. A grocer bought 8 loads of flour, averaging 3 tons each, at $2.60 per hundredweight. How much did it all cost him?

Solution: (1) Wt. of 1 load = 3 T. (Basis.)
8 × (1) = (2) Wt. of 8 loads = 24 T.
(3) 1 T. = 20 cwt. (Basis.)
24 × (3) = (4) 24 T. = 480 cwt.
(5) Cost of 1 cwt. = $2.60. (Basis.)
480 × (5) = (6) Cost of 480 cwt. = $1248. (Ans.)

EXERCISE LIII.

1. Reduce 42 pounds to ounces.
2. Reduce 7 hundredweight to ounces.
3. Reduce 1 ton 75 pounds to ounces.
4. Reduce 4 tons 14 hundredweight to pounds.
5. Reduce 18 tons 6 hundredweight 37 pounds 12 ounces to ounces.
6. A grocer bought 24 tubs of butter, of 2 hundredweight each, at 20¢ per pound. How much did it all cost?
7. A merchant bought 4 sacks of sugar, containing 1 hundredweight each, paying $7 a sack. He sold the sugar at 10¢ a pound. What was his profit?
8. A man bought 14 head of hogs, averaging 3 hundredweight, at 5¢ per pound. Find the cost.

57. Table of Time Measures.—These measures are used in giving the time of the day, and also in counting the interest on notes, bills, etc. *Commit the table to memory.*

TABLE.

1 year (yr.)=12 months (mo.).
1 mo.=30 days (da.).
1 da.=24 hours (hr.).
1 hr.=60 minutes (min.).
1 min.=60 seconds (sec.).

Many facts about the *exact year, leap year, weeks, days in the calendar months,* and the like, are omitted here. They may be given orally or omitted altogether until the pupil takes up the advanced work. Enough is given here to answer all the purposes of this work.

EXERCISE LIV.

1. How many minutes in 1 day?
2. How many seconds in 1 day?
3. Reduce 5 years 4 months 15 days to days.
4. Reduce 14 years 7 months to months.
5. Reduce 2 years 8 months 23 days to days.
6. Reduce 5 years 3 months 17 days to days.
7. Reduce 7 years 9 months 3 days to days.
8. Reduce 3 years 11 months 29 days to days.

58. Table of Linear Measures.—These measures are used in measuring heights, lengths, widths, distances, etc. *Commit the table to memory.*

TABLE.

1 yard (yd.)=3 feet (ft.).
1 ft.=12 inches (in.).

EXERCISE LV.

1. Reduce 44 yards to inches.
2. Reduce 320 feet to inches.
3. Reduce 17 yards 2 feet to inches.
4. How many inches in 54 yards 1 foot 7 inches?
5. I buy 5 yards of wire netting at 5¢ per foot. What does it cost me?

V. DIVISION.

59. Definitions.—Division is the process of finding how many times one number contains another. The number contained is called the **Divisor.** The number containing the divisor is called the **Dividend.** The result of division is called the **Quotient.**

60. Remainders.

$$14=4+4+4+2.$$

14 units contains 4 units 3 times, and has 2 units more. Then,

$$14\div4=3, \text{ remainder } 2.$$

EXERCISE LVI.

Call quotients and remainders quickly:

1.	*2.*	*3.*	*4.*
$10\div3$	$33\div8$	$60\div8$	$70\div9$
$11\div5$	$40\div9$	$57\div9$	$75\div8$
$14\div5$	$25\div6$	$46\div5$	$67\div8$
$17\div4$	$30\div7$	$52\div7$	$55\div8$
$18\div4$	$37\div7$	$38\div6$	$48\div7$
$15\div4$	$44\div6$	$46\div4$	$61\div9$
$19\div4$	$47\div8$	$23\div7$	$43\div5$
$28\div6$	$50\div7$	$35\div8$	$78\div9$

61. Dividing, when the Divisor is Smaller than Ten.—Put the divisor on the left of the dividend. Begin at the left of the dividend to divide. Write the quotient below.

EXAMPLES.

1. Divisor. 3)93 Dividend. 31 Quotient.

Explanation: (1) $9\div3=3$. Write the *3* below the *9*.

(2) $3\div3=1$. Write the *1* below the *3* of the dividend. *Quotient 31.*

2. $4)\overline{76}$ $\quad 19$

Explanation: (1) $7 \div 4 = 1$, and 3 remaining. Write the *1* below the *7*.

(2) The remainder 3 is tens. 3 tens + 6 units = 36 units. $36 \div 4 = 9$. Write the *9* below. *Quotient 19.*

3. $8)\overline{169}$ $\quad 21$ rem. 1

Explanation: (1) 1 of the dividend is smaller than 8, then we use 16.

(2) $16 \div 8 = 2$. Write the *2* below the *6*.

(3) $9 \div 8 = 1$; rem., 1. Write the quotient *1* to the right of the *2*, and the *rem. 1* below. *Quotient 21, remainder 1.*

4. $6)\overline{7220}$ $\quad 1203$ rem. 2

Explanation: (1) $7 \div 6 = 1$, rem. 1. Write the quotient *1* below the *7*.

(2) Remainder 1 is 1 thousand. 1 thousand + 2 hundred = 12 hundred. $12 \div 6 = 2$. Write the *2* below.

(3) $2 \div 6 = 0$, rem. 2. Write *0* below.

(4) The remainder 2 tens = 20 units. $20 \div 6 = 3$, rem. 2. Write the *3* at the right of the *0*, and the *rem. 2* below. *Quotient 1203, remainder 2.*

5.	*6.*	*7.*	*8.*
$5)\overline{78}$	$4)\overline{64}$	$8)\overline{175}$	$9)\overline{359}$
$\quad 15$	$\quad 16$	$\quad 21$	$\quad 39$
rem. 3		rem. 7	rem. 8

NOTE.—Make the above examples plain to the pupil, before taking up the following exercise.

EXERCISE LVII.

Find quotients and remainders:

1.	*2.*	*3.*	*4.*	*5.*
$6)\overline{56}$	$6)\overline{266}$	$6)\overline{432}$	$6)\overline{360}$	$6)\overline{750}$
6.	*7.*	*8.*	*9.*	*10.*
$5)\overline{59}$	$5)\overline{175}$	$5)\overline{345}$	$5)\overline{581}$	$5)\overline{673}$
11.	*12.*	*13.*	*14.*	*15.*
$7)\overline{68}$	$7)\overline{174}$	$7)\overline{193}$	$7)\overline{172}$	$7)\overline{188}$

16.	*17.*	*18.*	*19.*	*20.*
8)328	8)240	8)407	8)204	8)200
21.	*22.*	*23.*	*24.*	*25.*
9)504	9)324	9)567	9)431	9)234
26.	*27.*	*28.*	*29.*	*30.*
3)5001	7)7048	9)1234	8)1234	6)1234
31.	*32.*	*33.*	*34.*	*35.*
5)8460	6)3942	4)4270	8)9908	9)11074

Give process and results orally:

36. 52÷2	*51.* 59÷7	*66.* 84÷5
37. 43÷6	*52.* 67÷4	*67.* 85÷3
38. 64÷4	*53.* 95÷5	*68.* 83÷5
39. 45÷3	*54.* 64÷8	*69.* 88÷6
40. 36÷5	*55.* 73÷9	*70.* 82÷9
41. 47÷7	*56.* 38÷4	*71.* 94÷8
42. 48÷8	*57.* 82÷7	*72.* 93÷7
43. 49÷7	*58.* 63÷7	*73.* 86÷4
44. 52÷4	*59.* 78÷6	*74.* 97÷3
45. 83÷5	*60.* 66÷4	*75.* 94÷7
46. 54÷6	*61.* 74÷8	*76.* 96÷8
47. 55÷5	*62.* 73÷9	*77.* 95÷5
48. 58÷4	*63.* 77÷6	*78.* 98÷9
49. 27÷3	*64.* 71÷5	*79.* 99÷6
50. 51÷5	*65.* 78÷7	*80.* 100÷8

NOTE.—The process should be followed out orally. In Ex. 36 say, "2 in 5, twice; in 12, 6 times. Result, 26."

Solve orally:

81. How many hats at $3 each can I buy for $45?

Solution: I can buy as many hats as $3 is contained times in $45, or 15 hats.

82. How many bushels in 136 pecks?

83. How many lengths of a 6-foot pole will measure a distance of 342 feet?

84. I can buy blackberries at 8¢ per quart. How many quarts can I get for $1.92?

NOTE.—Consider the $1.92 as 192¢.

85. How many hours must a boy work to earn $1.62 at 9¢ per hour?

86. At $8 each, how many calves can be bought for $248?

87. $2.35 can be changed into how many 5-cent pieces?

88. At $9 each, how many tables can be bought for $342?

89. How many 7-bushel boxes would be needed to hold 126 bushels of grain?

90. How many rows of opera-chairs will be needed to seat 132 persons, if there are 6 chairs in a row?

62. Dividing, when the Divisor is Larger than Nine.—

Write the quotient above the dividend.

EXAMPLES.

1.

Divisor. 13 Quotient.
14)182 Dividend.
14
42
42

Explanation: (1) 18 ÷ 14 = 1 (with some remainder). Write the *1* above the *8*. 1 × 14 = 14. Write the product below the *18*. Subtract, and to the remainder 4 bring down the next figure of the dividend, 2. (2) 42 ÷ 14 = 3. Write the *3* above. 3 × 14 = 42. Write the product below the 42 of the dividend, and subtract. *Quotient 13.*

2.

27
17)464
34
124
119
5 rem.

Explanation: (1) 46 ÷ 17 = 2 +.* 2 × 17 = 34. 46 − 34 = 12. Bring down 4. 124 ÷ 17 = 7 +. 7 × 17 = 119. 124 − 119 = 5. *Quotient 27, remainder 5.*

* Read, 2+, "*two plus.*" It here means *2 and a remainder.*

Since we must now deal with large divisors, it is not always easy to tell just what the quotient figure should be without trying. The following examples will show how to find the quotient figure by trial.

3.

$$\begin{array}{r} 15 \\ 23\overline{)387} \\ 23 \\ \hline 157 \\ 115 \\ \hline 42 \end{array}$$

Explanation: Suppose we do not know how many times 23 is contained in 157, and we try *5 times*. $5 \times 23 = 115$. $157 - 115 = 42$. But 42 is larger than 23; therefore, 157 will contain 23 more than 5 times.

REMEMBER: *When the remainder is larger than the divisor the quotient figure is* **too small.**

4.

$$\begin{array}{r} 17 \\ 23\overline{)387} \\ 23 \\ \hline 157 \\ 161 \end{array}$$

Explanation: If we do not know how many times 157 contains 23, suppose we try *7 times*. $7 \times 23 = 161$. But 161 is larger than 157; therefore, 157 will not contain 23 7 times.

REMEMBER: *When the number to be subtracted is larger than the number from which you are to subtract, the quotient figure is* **too large.**

EXERCISE LVIII.

Answer quickly. Is the quotient figure right? too large? or too small?

1.

$$\begin{array}{r} 2 \\ 26\overline{)78} \\ 52 \\ \hline 26 \end{array}$$

2.

$$\begin{array}{r} 5 \\ 37\overline{)162} \\ 185 \end{array}$$

3.

$$\begin{array}{r} 3 \\ 22\overline{)79} \\ 66 \\ \hline 13 \end{array}$$

4.

$$\begin{array}{r} 5 \\ 94\overline{)465} \\ 470 \end{array}$$

5.

$$\begin{array}{r} 4 \\ 32\overline{)130} \\ 128 \\ \hline 2 \end{array}$$

6.

$$\begin{array}{r} 3 \\ 34\overline{)97} \\ 102 \end{array}$$

7.

$$\begin{array}{r} 4 \\ 17\overline{)88} \\ 68 \\ \hline 20 \end{array}$$

8.

$$\begin{array}{r} 8 \\ 13\overline{)107} \\ 104 \\ \hline 3 \end{array}$$

Find the right quotient figure:

9. 144÷19	*13.* 346÷55	*17.* 596÷95
10. 183÷24	*14.* 783÷92	*18.* 304÷38
11. 99÷12	*15.* 183÷49	*19.* 103÷23
12. 130÷15	*16.* 476÷81	*20.* 706÷87

63. Dividing, when the Divisor is Larger than Nine. (Continued.)

EXAMPLES.

1.

```
      308
  15)4620
     45
     120
     120
```

Explanation: After bringing down the "2," the 12 was too small to contain the 15. Put a *0* in the quotient, and bring down the next figure.

2.

```
       17
  2x00)34x00
```

Explanation: Cut off all 0's found on the right of the divisor, and an equal number of figures from the right of the dividend. Divide the remaining part of the dividend by the remaining part of the divisor. *Quotient, 17.*

3.

```
       17
  2x00)34x05
```

5 rem.

Explanation: The 5 cut off is remainder.

4.

```
        2
  17x00)35x05
       34
      105 rem.
```

Explanation: If there be a remainder from the dividing, the part of the dividend cut off is annexed to form the complete remainder.

5.

```
       34
  1x000)34x050
```

50 rem.

Explanation: When the divisor is 10, 100, 1000, etc., the part cut off of the dividend is remainder, and the part not cut off is quotient.

EXERCISE LIX.

Find quotients and remainders:

1. 143÷11	*11.* 675÷27	*21.* 1260÷12
2. 312÷13	*12.* 504÷21	*22.* 6359÷13
3. 468÷13	*13.* 810÷45	*23.* 4835÷15
4. 204÷12	*14.* 1586÷38	*24.* 7032÷16
5. 330÷15	*15.* 575÷23	*25.* 4350÷30
6. 612÷18	*16.* 2400÷32	*26.* 5700÷20
7. 704÷16	*17.* 2310÷66	*27.* 9084÷40
8. 270÷18	*18.* 1932÷23	*28.* 7350÷50
9. 475÷19	*19.* 3066÷42	*29.* 8520÷60
10. 540÷20	*20.* 8400÷70	*30.* 7400÷70

31. 6250÷80	*41.* 37902÷ 500
32. 7920÷90	*42.* 73054÷1000
33. 3442÷22	*43.* 49200÷1200
34. 4536÷21	*44.* 84360÷ 980
35. 5184÷36	*45.* 92080÷ 900
36. 30885÷71	*46.* 83725÷ 346
37. 21050÷62	*47.* 44550÷ 594
38. 83352÷92	*48.* 31820÷ 740
39. 54250÷31	*49.* 35408÷ 984
40. 63168÷47	*50.* 30240÷1260

Find answers:

51. How many loads of 36 bushels each can be taken from a bin containing 504 bushels of wheat?

52. How many fields of 40 acres each can be made out of a section (640 acres) of land?

53. A fair association has $650 to offer in prizes. How many $50 prizes can the association offer?

54. A train carries 484 people. How many coaches are required, if 44 persons fill a coach?

55. How many boxes will be required for packing 598 books, if 46 books will fill a box?

64. Three Applications of Division.

$$9\overline{)36}\quad 4$$

The fact of this division is that 36 things contain 9 of those things *4 times*.

NOTE.—We must be careful not to fall into the error of thinking that 36 *contains* 9 as a bucket *contains* water. The word "*contains*," as here used, means "*is made up of*," or "*is composed of*."

FIRST APPLICATION.—*Problem:* There are 36 apples in a lot. How many times can we take 9 apples from the lot?

Answer: From division we know that *36 apples contain 9 apples 4 times*. Then, we can take 9 apples from the lot *4 times*.

This is the most *direct application* of division, and the only one which we have presented so far in this part.

SECOND APPLICATION.—*Problem:* There are 36 apples in one lot and 9 apples in another. The first lot is how many times as large as the second lot?

Answer: From division, we know that *36 apples contain 9 apples 4 times*. Then, 36 apples must be *4 times* as large as 9 apples of the same size.

This is the *application* nearest like the direct application, and may be called the **Ratio Idea of Division.**

THIRD APPLICATION.—*Problem:* There are 36 apples in a lot. What is $\frac{1}{9}$ of the lot?

Answer: From division, we know that *36 apples contain 9 apples 4 times*. Suppose we take 9 of the 36 apples, and with them begin 9 new lots; for every time we can take 9 apples from the 36 apples, we can put 1 apple in each new lot. But 36 apples contain 9 apples *4 times*. Therefore we can put 4 apples in each new lot. Thus, we have separated 36 apples

into 9 equal new lots, and one lot contains $\frac{1}{9}$ of 36 apples or 4 apples.

NOTE.—If necessary, the teacher should illustrate this with objects.

This may be called the **Fraction Idea of Division.**

65. Different Ways of Expressing Division.—There are many ways or methods of expressing division. The following are in common use:

(1) $36 \div 9$

This expression is usually read, "*36 divided by 9.*" It means (1) 36 contains 9, (2) the ratio of 36 to 9, or (3) $\frac{1}{9}$ of 36. It is a general method of expressing division.

(2) $\frac{36}{9}$

This expression may be read, (1) "*36 divided by 9,*" (2) "*The ratio of 36 to 9,*" or (3) "*36 ninths.*" It, like the first expression, is a general method of expressing division.

(3) 36 : 9

Read, "*The ratio of 36 to 9.*" This expresses only the Ratio Idea of Division.

(4) $\frac{1}{9}$ of 36

Read, "*One-ninth of 36.*" This expresses only the Fraction Idea of Division.

66.—Disposing of the Remainder.

EXAMPLES.

1. Divide 134 by 5.

$$5)\overline{134}$$
$$\overline{26\frac{4}{5}}$$

Explanation: After we had obtained the quotient 26, we had a remainder of 4. This 4 is to be divided by 5. But we learned in Expression (2), Article 65, that 4 divided by 5 may be put in the form, $\frac{4}{5}$, and called "*four-fifths.*" This we

put on the right of the *26* and call the expression, $26\frac{4}{5}$, "*twenty-six and four-fifths.*"

2. Find $\frac{1}{24}$ of 373.

$$\begin{array}{r} 15\frac{13}{24} \\ 24)373 \\ 24 \\ \hline 133 \\ 120 \\ \hline 13 \end{array}$$

Read the quotient, "*15 and 13 twenty-fourths.*"

EXERCISE LX.

Give results orally:

1.	*2.*	*3.*	*4.*
$17 \div 6$	$78 : 7$	$\frac{1}{5}$ of 48	$\frac{34}{5}$
$19 \div 8$	$65 : 9$	$\frac{1}{8}$ of 59	$\frac{29}{7}$
$13 \div 4$	$54 : 7$	$\frac{1}{7}$ of 80	$\frac{85}{9}$
$16 \div 7$	$39 : 8$	$\frac{1}{6}$ of 37	$\frac{68}{9}$
$23 \div 5$	$88 : 3$	$\frac{1}{9}$ of 80	$\frac{87}{8}$
$34 \div 9$	$47 : 6$	$\frac{1}{4}$ of 49	$\frac{73}{5}$

Prepare work in writing:

5. $346 \div 17$	*13.* $\frac{1}{24}$ of 347
6. $428 \div 19$	*14.* $\frac{1}{57}$ of 617
7. $651 \div 25$	*15.* $\frac{1}{49}$ of 509
8. $438 \div 29$	*16.* $\frac{1}{23}$ of 730
9. $566 : 27$	*17.* $\frac{863}{30}$
10. $406 : 21$	*18.* $\frac{593}{45}$
11. $333 : 56$	*19.* $\frac{806}{63}$
12. $500 : 39$	*20.* $\frac{999}{64}$

67. Taking a Part of an Equation.—*We may take a part of an equation, and the result will be an equation, if we take the same part of each member of the equation.*

EXAMPLES.

1. 4 pecks = 1 bushel. Find $\frac{1}{4}$ of it.

Process: (1) 4 pk. = 1 bu.
$\frac{1}{4}$ of (1) = (2) 1 pk. = $\frac{1}{4}$ bu.

2. Cost of 9 hats = \$36. Find $\frac{1}{9}$ of it.

Process: (1) Cost of 9 hats = \$36.
$\frac{1}{9}$ of (1) = (2) Cost of 1 hat = \$4.

3. \$25 = price of 50 books. Find $\frac{1}{25}$ of it.

Process: (1) \$25 = price of 50 books.
$\frac{1}{25}$ of (1) = (2) \$1 = price of 2 books.

4. Weight of 30 loads of wheat = 62400 pounds. Find $\frac{1}{30}$ of it.

Process: (1) Wt. of 30 loads = 62400 lb.
$\frac{1}{30}$ of (1) = (2) Wt. of 1 load = 2080 lb.

EXERCISE LXI.

1. Price of 7 mats = \$21. Find $\frac{1}{7}$ of it.
2. Price of 40 lb. of sugar = \$4.40. Find $\frac{1}{40}$ of it.
3. \$6 = cost of 30 melons. Find $\frac{1}{6}$ of it.
4. 8 quarts = 1 peck. Find $\frac{1}{8}$ of it.
5. 1760 yards = 1 mile. Find $\frac{1}{1760}$ of it.
6. 80 chains = 1 mile. Find $\frac{1}{80}$ of it.
7. \$25 = cost of 15 books. Find $\frac{1}{5}$ of it.
8. Cost of 10 cords of wood = \$30. Find $\frac{1}{10}$ of it.
9. Travel of light for 4 seconds = 744000 miles. Find $\frac{1}{4}$ of it.
10. Cost of 12 shares of railroad stock = \$6000. Find $\frac{1}{12}$ of it.

68. Problems.

EXAMPLES.

1. A man spent \$3520 for horses, at \$55 each. How many did he buy?

Solution: (1) \$55 = cost of 1 horse. (Basis.)
$\frac{1}{55}$ of (1) = (2) \$1 = cost of $\frac{1}{55}$ horse.
3520 × (2) = (3) \$3520 = cost of $\frac{3520}{55}$ horses, or 64 horses. (Ans.)

NOTE.—In (2), we get the "cost of $\frac{1}{55}$ horse." Such an expression is mathematically correct, but some may take exception to it on the ground that in business we have no such thing as "cost of $\frac{1}{55}$ horse." Those who prefer it may use the following solution:

Solution: (1) \$55 = cost of 1 horse. (Basis.)
$\frac{1}{55}$ of (1) = (2) \$1 = $\frac{1}{55}$ of cost of 1 horse.
3520 × (2) = (3) \$3520 = $\frac{3520}{55}$ of cost of 1 horse = 64 × cost of 1 horse = cost of 64 horses. (Ans.)

2. Two men measure a distance of 5280 feet with a chain 66 feet long. How many lengths of the chain in that distance?

Solution: (1) 66 ft. = 1 ch. (Basis.)
$\frac{1}{66}$ of (1) = (2) 1 ft. = $\frac{1}{66}$ ch.
5280 × (2) = (3) 5280 ft. = $\frac{5280}{66}$ ch., or 80 ch. (Ans.)

3. A man buys 30 books for \$180. At that rate what would 7 books cost?

Solution: (1) Cost of 30 books = \$180. (Basis.)
$\frac{1}{30}$ of (1) = (2) Cost of 1 book = \$6.
7 × (2) = (3) Cost of 7 books = \$42. (Ans.)

4. In how many days will a horse travel 420 miles, at 28 miles per day?

Solution: (1) 28 miles = travel for 1 day. (Basis.)
$\frac{1}{28}$ of (1) = (2) 1 mile = travel for $\frac{1}{28}$ day.
420 × (2) = (3) 420 miles = travel for 15 days. (Ans.)

EXERCISE LXII.

1. How many loads of 45 bushels each in 1170 bushels of wheat?

2. There are 16 ounces in 1 pound. How many pounds in 1964 ounces?

3. There are 60 pounds of wheat in a bushel. How many bushels in 7200 pounds?

4. I can buy steers at \$18 each. How many can I buy for \$612?

5. If town lots cost \$140 each, how many can be bought for \$1680?

6. How many bins containing 248 bushels each will be needed to hold 8928 bushels?

7. How many acres of 160 sq. rd. each in a square mile of 102400 sq. rd.?

69. Measures and Weights Again.—When we were changing, or "reducing," numbers from higher (larger) to lower (smaller) units, the tables were given in one form. Now, we are going to change numbers from lower to higher units. Notice carefully the changes in the tables.

DRY MEASURES.

1 pt. = $\frac{1}{2}$ qt.
1 qt. = $\frac{1}{8}$ pk.
1 pk. = $\frac{1}{4}$ bu.

EXAMPLES.

1. Reduce 128 pints to bushels.

Solution: (1) 1 pt. = $\frac{1}{2}$ qt. (Basis.)
128 × (1) = (2) 128 pt. = $\frac{128}{2}$ qt. = 64 qt.
(3) 1 qt. = $\frac{1}{8}$ pk. (Basis.)
64 × (3) = (4) 64 qt. = $\frac{64}{8}$ pk. = 8 pk.
(5) 1 pk. = $\frac{1}{4}$ bu. (Basis.)
8 × (5) = (6) 8 pk. = 2 bu. (Ans.)

2. Reduce 315 pints to higher units.

Solution: (1) 1 pt. = $\frac{1}{2}$ qt. (Basis.)
315 × (1) = (2) 315 pt. = $\frac{315}{2}$ qt. = $157\frac{1}{2}$ qt. = 157 qt. 1 pt.
(Reduce the quarts to pecks.)
(3) 1 qt. = $\frac{1}{8}$ pk. (Basis.)
157 × (3) = (4) 157 qt. = $\frac{157}{8}$ pk. = 19 pk. 5 quarts.
(Reduce the pecks to bushels.)
(5) 1 pk. = $\frac{1}{4}$ bu. (Basis.)
19 × (5) = (6) 19 pk. = $\frac{19}{4}$ bu. = 4 bu. 3 pk.
∴ 315 pints = 4 bu. 3 pk. 5 qt. 1 pt. (Ans.)

EXERCISE LXIII.

1. Reduce 96 quarts to bushels.

2. Reduce 129 pints to higher units.

3. Reduce 215 pints to higher units.

4. Reduce 408 pints to higher units.

5. Reduce 377 pints to higher units.

6. By referring to Article 55, complete the following table of *Liquid Measures:*

1 gi. = $\frac{1}{4}$ pt.
1 pt. = ( ) qt.?
1 qt. = ( ) gal.?

7. Reduce 320 gills to quarts.

8. Reduce 75 pints (liquid) to higher units.

9. Reduce 960 gills to gallons.

10. Reduce 734 gills to higher units.

11. By referring to Article 56, complete the following table of *Avoirdupois Weights:*

1 oz. = $\frac{1}{16}$ ℔.
1 ℔. = ( ) cwt.?
1 cwt. = ( ) T.?

12. Reduce 8048 ounces to higher units.

13. Reduce 160960 ounces to higher units.

14. Reduce 3450 ounces to higher units.

15. Reduce 44444 pounds to higher units.

16. I sold 4 hogs at $4.50 per hundred. The first hog weighed 284 pounds; the second, 346 pounds; the third, 272 pounds; and the fourth, 298 pounds. How much did the hogs bring?

17. By referring to Article 57, complete the following table of *Time Measures:*

1 sec. = $\frac{1}{60}$ min.
1 min. = ( ) hr.?
1 hr. = ( ) da.?
1 da. = ( ) mo.?
1 mo. = ( ) yr.?

18. Reduce 172800 seconds to days.

19. Reduce 172800 minutes to months.

20. Reduce 1080 days to years.

21. Reduce 540 minutes to hours.

22. Reduce 8445 minutes to higher units.

23. Reduce 439060 seconds to higher units.

24. By referring to Article 58, complete the following table of *Linear Measures:*

1 in.=( ) ft.?
1 ft.=( ) yd.?

25. Reduce 36 inches to yards.

26. Reduce 1008 inches to feet.

27. Reduce 731 inches to higher units.

28. Reduce 528 feet to yards.

29. Reduce 5090 inches to yards.

30. How much is a bolt of muslin 1548 inches long worth, at 10¢ per yard?

VI. DERIVED OPERATIONS.

A. CLASSIFICATION OF NUMBER.

70. Definitions.—A **Number** is one or more units, considered as forming one quantity or amount.

A **Unit** is a single thing or one. Units are classed as *concrete* or *abstract; integral* or *fractional.*

A **Concrete Unit** is one *thing.* As,

1 book, 1 man, 1 box.

An **Abstract Unit** is simply *one*, a *ratio*, and not a *thing.* As,

1.

A unit, not considered as a part of any other unit, is an **Integral Unit.** As,

1 book, $1, 1 foot.

A unit considered as a part of some other unit is a **Fractional Unit.** As,

$\frac{1}{5}$ foot, \$$\frac{1}{4}$, $\frac{1}{8}$ pint.

NOTE.—The $\frac{1}{5}$ is 1 fifth of 1 foot. Then, the unit, $\frac{1}{5}$, is a part of the unit, 1 foot; so, the $\frac{1}{4}$ is a part of \$1, and the $\frac{1}{8}$ is a part of 1 pint.

A number composed of *concrete units* is a **Concrete Number.** As,

150 men, \$744, 280 miles.

A number composed of *abstract units* is an **Abstract Number.** As,

150, 744, 280.

When a concrete number and an abstract number have each the same number of units, they are called *Corresponding* numbers. *Every concrete number has its corresponding abstract number.* Thus,

150 men, 150,
\$744, 744,
280 miles, 280.

A number composed of *integral units* is an **Integral Number,** or an **Integer.** As,

346 books, 281.

NOTE.—An Integer may be either concrete or abstract.

A number composed of *fractional units* is a **Fractional Number,** or a **Fraction.** As,

\$$\frac{3}{5}$, $\frac{5}{13}$.

NOTE.—A Fraction may be either concrete or abstract.

A number composed partly of *integral units* and partly of *fractional units* is a **Mixed Number.** As,

\$$5\frac{1}{2}$, $6\frac{3}{5}$.

NOTE.—A Mixed Number may be either concrete or abstract.

An integer is a **Prime Number** when it cannot be formed by multiplying any two other integers together. As,

1, 2, 3, 5, 7, 11, 13, 17.

An integer is a **Composite Number** when it can be formed by multiplying two or more other integers together. As, 4, 6, 8, 9, 10, 12, 14, 15.

EXERCISE LXIV.

Write:

1. Ten Integers.
2. Ten Fractions.
3. Ten Mixed Numbers.
4. Ten Abstract Numbers.
5. Ten Concrete Numbers.
6. Ten Prime Numbers.
7. Ten Composite Numbers.

B. FACTORING.

71. Definitions.—We have learned that every composite number may be formed by multiplying two or more other numbers together. **Factors** of a number are the numbers which, multiplied together, form that number. As,

$$6\times7\times5=210.$$

6, 7 and 5 are factors of 210.

A prime number has but two integral factors, *itself* and *one*. Thus,

$$1\times7=7$$

1 and 7 are factors of 7.

NOTE.—Since *unity* and the *number itself* are factors of any number, they are seldom mentioned in giving factors.

A **Prime Factor** is a prime number that is a factor. A **Composite Factor** is a composite number that is a factor.

A **Common Factor** of two or more numbers is a number that is a factor of all of them. Numbers that have no common factor (except unity) are *prime to each other.*

A number can have but *one set of prime factors;* but a number that has 3 or more prime factors (besides unity) may have

more than one set of factors, some of which are composite. Thus,

$$2\times3\times5=30.$$

$$30=6\times5=10\times3=2\times15.$$

30 has one set of prime factors: 2, 3, 5; but three sets of factors, some of which are composite: 6, 5; 10, 3; and 2, 15.

72. Factoring by Inspection.—Since a *multiplier* or a *multiplicand* is a *factor* of the product, the factors may be found by *Inspection*, when the product is small.

EXAMPLES.

1. Find the factors of 4.

Think: $4=2\times2$. *Call:* "Factors, 2, 2."

2. Find the prime factors of 12.

Think: $12=2\times2\times3$. *Call:* "Factors, 2, 2, 3."

3. Find the prime factors of 42.

Think: $42=2\times3\times7$. *Call:* "Factors, 2, 3, 7."

EXERCISE LXV.

Give the prime factors orally:

1. 4	13. 24	25. 49
2. 6	14. 25	26. 50
3. 9	15. 26	27. 54
4. 10	16. 28	28. 56
5. 12	17. 30	29. 60
6. 14	18. 32	30. 63
7. 15	19. 35	31. 64
8. 16	20. 36	32. 72
9. 18	21. 40	33. 84
10. 20	22. 42	34. 90
11. 21	23. 44	35. 96
12. 22	24. 48	36. 100

73. Factoring by Division.—When there is no remainder, the divisor and quotient are **Factors** of the dividend. When the number to be factored is large, we employ division in finding its factors. But before we can divide we must find a divisor by inspection. This may usually be done by aid of the following *principles*. *Commit them to memory.*

PRINCIPLES.

I. One factor is 2, if the number ends in 0, 2, 4, 6, 8.

II. One factor is 3, if 3 is a factor of the sum obtained by adding the digits of the number.

III. One factor is 5, if the number ends in 0, 5.

Illustrations: I. One factor of 878 is 2 (Prin. I), because the number ends in 8.

II. One factor of 2571 is 3 (Prin. II), because the sum of the digits is 15, and 3 is a factor of 15.

III. One factor of 875 is 5 (Prin. III), because the number ends in 5.

EXAMPLES.

1. Find the prime factors of 210.

Process:

5)210
3) 42
2) 14
7

Explanation: (1) By Prin. III, 5 is a factor of 210.

(2) By Prin. II, 3 is a factor of 42.

(3) By Prin. I, 2 is a factor of 14.

∴ 5, 3, 2, and 7 are the factors of 210.

2. Find the prime factors of 5040.

2)5040
2)2520
2)1260
2) 630
5) 315
3) 63
3) 21
7

Factors, 2, 2, 2, 2, 5, 3, 3, 7.

EXERCISE LXVI.

Find the prime factors of the following numbers:

1. 120	*5.* 840	*9.* 960	*13.* 324	*17.* 5760
2. 540	*6.* 168	*10.* 312	*14.* 512	*18.* 2340
3. 630	*7.* 104	*11.* 624	*15.* 1050	*19.* 3024
4. 600	*8.* 196	*12.* 225	*16.* 1560	*20.* 8320

74. Cancellation.—Sometimes the dividend and divisor have the same (common) factors. When this is so, the division may often be made easier and shorter by omitting or canceling out the common factors from both dividend and divisor. For cancellation, we usually place the dividend above and the divisor below a horizontal line.

EXAMPLES.

NOTE.—The first four examples are given to develop the principle of cancellation. It is intended that the pupil use the developed plan as given in the fifth and sixth examples.

1. Divide 48 by 16.

$$\frac{\text{Dividend}\ldots 48 = \cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}\times 3}{\text{Divisor}\ldots. 16 = \cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}} = 3.$$

2. Divide 140 by 35.

$$\frac{140 = 2\times 2\times\cancel{5}\times\cancel{7}}{35 = \quad\cancel{7}\times\cancel{5}} = 4.$$

Sometimes the dividend or divisor, or both dividend and divisor, are expressed in the form of continued multiplication. Then it is a special help to use cancellation.

3. Divide 60×25 by 5×30.

$$\frac{60\times 25 = 2\times\cancel{2}\times\cancel{3}\times\cancel{5}\times\cancel{5}\times 5}{5\times 30 = \quad\cancel{5}\times\cancel{2}\times\cancel{3}\times\cancel{5}} = 10.$$

It is not necessary to use prime factors if composite factors are common.

4. Divide 30×72 by 15×9.

$$\frac{30\times72}{15\times\ 9}=\frac{2\times\cancel{15}\times8\times\cancel{9}}{\cancel{15}\times\cancel{9}}=16.$$

Instead of rewriting and factoring before canceling, you can do this work mentally.

5. Divide 50×96 by 15×8.

$$\frac{\overset{10}{\cancel{50}}\times\overset{4}{\cancel{96}}}{\underset{3}{\cancel{15}}\times\cancel{8}}=40.$$

Explanation: (1) Cancel 5 out of 50 and 15, leaving 10 above and 3 below. (2) 96 contains 8 12 times; cancel 8 and 96, placing 12 above. (3) 12 contains 3 4 times; cancel 3 and 12, and place 4 above. (4) $4\times10=40$.

6. How many times will $48\times27\times15$ contain $9\times6\times10$?

$$\frac{\overset{4}{\cancel{8}}\quad 3\quad 3}{\cancel{48}\times\cancel{27}\times\cancel{15}}{\cancel{9}\times\cancel{6}\times\underset{2}{\cancel{10}}}=36.$$

Explanation: (1) 6 in 48 8 times. Cancel 6 and 48, and place 8 above. (2) 9 in 27 3 times. Cancel, and write 3 above. (3) 5 is common to 15 and 10. Cancel, and write 3 above and 2 below. (4) 2 in 8 4 times. Cancel, and write 4 above. (5) $4\times3\times3=36$.

EXERCISE LXVII.

1. Divide 6×10 by 12.

2. Divide 9×24 by 36.

3. $\dfrac{12\times9\times5}{15\times9}=(\ \ )?$

4. $\dfrac{45\times18\times4}{9\times20}=(\ \ )?$

5. $\frac{30\times56\times80}{100\times12\times28}=(\quad)?$

6. $\frac{36\times50\times28}{70\times18\times10}=(\quad)?$

7. $\frac{72\times95\times48}{24\times19\times15}=(\quad)?$

8. $\frac{14\times26\times72}{13\times24\times21}=(\quad)?$

9. $\frac{36\times64\times100}{25\times16\times8}=(\quad)?$

10. $\frac{42\times52\times108}{78\times60}=(\quad)?$

11. $\frac{200\times39\times9\times21}{50\times12\times27}=(\quad)?$

12. $\frac{210\times340\times1260}{280\times25\times252}=(\quad)?$

C. GREATEST COMMON DIVISOR.

75. Definition and Process.—The **Greatest Common Divisor** of two or more numbers is the largest number that is contained in each of them an integral number of times. *The G. C. D. of two or more numbers is the product of all the prime factors common to the numbers.* When the numbers are small enough to be factored mentally, *factoring* should be employed in finding the G. C. D.

EXAMPLES.

1. Find the G. C. D. of 210 and 150.

Factors of 210 are 3, 2, 5, 7.
Factors of 150 are 3, 2, 5, 5.
3, 2 and 5 are common.
$\therefore$ $3\times2\times5=30$, G.C.D.

2. Find the G. C. D. of 55 and 77.

$55=11\times5$.
$77=7\times11$.
$\therefore$ 11, being the only common factor, is the G.C.D.

3. Find the G. C. D. of 24, 48, and 72.

$24=2\times2\times2\times3$.
$48=2\times2\times2\times2\times3$.
$72=2\times2\times2\times3\times3$.
2, 2, 2, 3 are common factors.
$\therefore$ $2\times2\times2\times3=24$, G.C.D.

4. Find the G. C. D. of 15, 45, and 60.

5)15, 45, 60
3) 3, 9, 12
1, 3, 4

3 and 5 are all the common factors.
∴ 3×5=15, G.C.D.

NOTE.—In this process, use only such divisors as will divide all the numbers.

EXERCISE LXVIII.

Find the G. C. D. of the following:

1. 10, 25, 50.
2. 14, 35, 63.
3. 21, 63, 105.
4. 36, 60, 84.
5. 30, 75, 90.
6. 48, 12, 56.
7. 120, 150, 210.
8. 48, 128, 80.
9. 70, 42, 84.
10. 12, 18, 36, 66.
11. 24, 56, 72, 96.
12. 39, 65, 104, 156.

D. LEAST COMMON MULTIPLE.

76. Definition and Process.—The **Least Common Multiple** of two or more numbers is the smallest number that contains each of them an integral number of times. *The L. C. M. of two or more numbers contains every prime factor of the several numbers the greatest number of times that it occurs in any one of them*

EXAMPLES.

1. Find the L. C. M. of 15, 12, and 10.

15=3×5.
12=3×2×2.
10=5×2.
The L.C.M. must contain 3 once, 5 once, and 2 twice.
∴ 3×5×2×2=60, L.C.M.

2. Find the L. C. M. of 10, 12, 24, and 30.

2)10, 12, 24, 30
2) 5, 6, 12, 15
5) 5, 3, 6, 15
3) 1, 3, 6, 3
1, 1, 2, 1

$2\times2\times5\times3\times2=120$, L. C. M.

NOTE.—In this process you may divide by any *prime* number that will divide two or more of the numbers. Numbers not containing the divisor an integral number of times must be brought down; as 5 and 15 in the second division above. When the final results are prime to each other, the continued product of all the *divisors* and final results (1's may be omitted) will be the L.C.M.

3. Find the L. C. M. of 63, 108, 28, and 84.

3)63, 108, 28, 84
3)21, 36, 28, 28
7) 7, 12, 28, 28
2) 1, 12, 4, 4
2) 1, 6, 2, 2
1, 3, 1, 1

$3\times3\times7\times2\times2\times3=756$, L.C.M.

EXERCISE LXIX.

Find the L. C. M. of the following:

1. 5, 12, 15.
2. 6, 12, 18.
3. 7, 21, 36.
4. 20, 14, 30.
5. 9, 12, 18.
6. 10, 25, 50.
7. 15, 21, 30.
8. 16, 32, 48.
9. 10, 15, 20.
10. 30, 40, 50.
11. 75, 50, 100.
12. 84, 96, 120.

VII. PROPORTION.

77. Definitions.

$\$10:\$2=5.$

In this equation, the expression, "\$10:\$2," is called an **Indicated Ratio,** because it *indicates* that the ratio of \$10

to $2 is to be found. The first term in an indicated ratio is called the **Antecedent**; the second term, the **Consequent.**

In this equation the "5" is called the **Numerical Ratio,** or simply the **Ratio,** because it is the number that expresses the ratio itself.

When two Indicated Ratios are equal to the same numerical ratio, they are equal to each other; such an equation is called a **Proportion.** Thus,

$$10:2=5, \text{ and}$$
$$20:4=5.$$
$$\text{Then, } 10:2=20:4.$$

The last expression is a proportion. Instead of the sign of equality, the **Sign of Proportion, ::,** is usually employed. Thus,

$$10:2::20:4$$

Read "*10 is to 2 as 20 is to 4.*" There are four terms in a proportion. The first and last terms are called **Extremes;** the other two terms are called **Means.**

78. Solving a proportion.—Solving a proportion is the process of finding any one term when the other three terms are given.

EXAMPLES.

1. In the proportion 10:2::20:4, suppose the last term missing. Find it.

Process: 10:2::20:( )?

$$\frac{2\times 20}{10}=4.$$

NOTE.—Here we multiplied the *two means* together and divided by the *given extreme.*

2. In the same proportion, suppose the first term missing. Find it.

Process: ( ):2::20:4?

$$\frac{2\times\overset{5}{\cancel{20}}}{\cancel{4}}=10.$$

NOTE.—Here we multiplied the *two means* together and divided by the *given extreme.*

Either extreme may be found by dividing the product of the means by the given extreme. (Commit.)

3. In the same proportion, suppose the second term missing. Find it.

Process: 10:( )::20:4?

$$\frac{\overset{2}{\cancel{10}}\times\cancel{4}}{\underset{\cancel{2}}{\cancel{20}}}=2.$$

NOTE.—Here we multiplied the *two extremes* together and divided by the *given mean.*

4. In the same proportion, suppose the third term missing. Find it.

Process: 10:2::( ):4?

$$\frac{10\times\overset{2}{\cancel{4}}}{\cancel{2}}=20.$$

NOTE.—Here we multiplied the *two extremes* together and divided by the *given mean.*

Either mean may be found by dividing the product of the extremes by the given mean. (Commit.)

EXERCISE LXX.

1. 12:25::( ):125?
2. ( ):32::60:8?
3. 13:12::78:( )?
4. 54:( )::72:4?
5. 400:300::16:( )?
6. 900:( )::45:80?
7. 25:4000::( ):1200?
8. ( ):1875::500:375?

79. Solving Problems by Proportion.

Suppose, (a) *Cost of 1 book* = *$3*.

Then, $40 \times (1) = $ (b) Cost of 40 books = $120.
$30 \times (1) = $ (c) Cost of 30 books = $90.
$20 \times (1) = $ (d) Cost of 20 books = $60.
$15 \times (1) = $ (e) Cost of 15 books = $45.
$5 \times (1) = $ (f) Cost of 5 books = $15.

NOTE.—The last five equations are all obtained from the first, and all depend upon the same fact—that 1 book costs $3. They are all of the same nature, having "*Cost of books*" for their first members and "*$*" for their second members.

Let us make a few comparisons:

1. Compare (b) and (d).

(b) Cost of 40 books = $120.
(d) Cost of 20 books = $60.
(1) 40 : 20 = 2.
(2) 120 : 60 = 2.
∴ 40 : 20 :: 120 : 60.

NOTE.—The numbers in the first members have the same ratio as the numbers in the second members.

2. Compare (c) and (f).

(c) Cost of 30 books = $90.
(f) Cost of 5 books = $15.
(1) 30 : 5 = 6.
(2) 90 : 15 = 6.
∴ 30 : 5 :: 90 : 15.

3. Compare (e) and (f).

(e) Cost of 15 books = $45.
(f) Cost of 5 books = $15.
(1) 15 : 5 = 3.
(2) 45 : 15 = 3.
∴ 15 : 5 :: 45 : 15.

These comparisons all bring out one *relation*, which is always true:

Relation: In two equations of the same nature, and depending upon the same condition, the ratio between the numbers in the first members is equal to the ratio between the numbers in the second members. (Commit.)

EXAMPLES.

1. If 10 books cost \$20, what will 35 books cost?

Solution: (1) Cost of 35 books = \$( )? (Question.)
(2) Cost of 10 books = \$20. (Basis.)
(3) 35 : 10 :: ( ) : 20? (Proportion.)

$$\frac{35 \times 20}{10} = 70.$$

∴ the required number is \$70.

2. If I can buy 15 hogs for \$75, how many can I buy for \$90?

Solution: (1) \$90 = cost of ( ) hogs? (Question.)
(2) \$75 = cost of 15 hogs. (Basis.)
(3) 90 : 75 :: ( ) : 15? (Proportion.)

$$\frac{90 \times 15}{75} = 18.$$

∴ the required number is 18 hogs.

3. Reduce 8 bushels to pecks.

Solution: (1) 8 bu. = ( ) pk.? (Question.)
(2) 1 bu. = 4 pk. (Basis.)
(3) 8 : 1 :: ( ) : 4? (Proportion.)

$$\frac{8 \times 4}{1} = 32.$$

∴ the required number is 32 pecks.

4. If 30 acres of land cost $450, how much was that per acre?

Solution: (1) Cost of 1 acre = $( )? (Question.)
(2) Cost of 30 acres = $450. (Basis.)
(3) 1 : 30 :: ( ) : 450? (Proportion.)

$$\frac{1 \times 450}{30} = 15.$$

∴ the required answer is $15.

EXERCISE LXXI.

1. If $84 be paid for 28 tons of hay, what should be paid for 35 tons?

2. What is the price of coal per ton, if 33 tons cost $132?

3. How many cows, at $43 a head, will $19608 buy?

4. What will 324 pounds of sugar cost, at 9¢ per pound?

5. If 110 men eat 22 loaves of bread at a meal, how many men will 75 loaves feed?

6. What are 60 bushels of corn worth, if 3 bushels are worth $2?

7. When flour is worth $4 per barrel, how many barrels can I get for $736?

8. $25 will buy 60 slates. How many will $15 buy?

9. Ten rods of fence cost $5. What will 3 miles of the same kind of fence cost?

NOTE.—First reduce 3 miles to rods. 1 mile = 320 rods.

10. 100 lb. = 1 cwt. Reduce 84300 lb. to cwt.

VIII. COMMON FRACTIONS.

80. Definitions.—A Fraction is a number composed of *Fractional Units.* Review Article 70.

What is a unit? Integral unit? Fractional unit?

A **Common Fraction** is a fraction expressed by two numbers, placed one above and the other below a horizontal line. Thus,

$\frac{5}{17}$, $\frac{346}{9}$.

The number below the line is called the **Denominator,** and tells the size of the fractional unit. The number above the line is called the **Numerator,** and tells the number of units in the fraction. The numerator and denominator are called the **Terms** of the fraction.

5 ....Numerator.
17..Denominator.

Explanation: The "*17*" tells that the units are *seventeenths;* the "*5*" tells that this fraction expresses *five* units—*five-seventeenths.*

A *common fraction* is also an expression of division. Review Article 65, (2).

When the numerator is equal to or larger than the denominator, the fraction is called an **Improper Fraction.**

EXERCISE LXXII.

Read, and give the number and size of units in each of the following fractions:

1. $\frac{1}{7}$	*6.* $\frac{15}{43}$	*11.* $\frac{17}{120}$
2. $\frac{3}{10}$	*7.* $\frac{48}{97}$	*12.* $\frac{243}{472}$
3. $\frac{7}{5}$	*8.* $\frac{34}{12}$	*13.* $\frac{800}{999}$
4. $\frac{12}{17}$	*9.* $\frac{1211}{132}$	*14.* $\frac{1}{1000}$
5. $\frac{9}{13}$	*10.* $\frac{349}{12}$	*15.* $\frac{9876}{500}$

81.—Reducing Fractions to Higher Terms.— *If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.* (Commit.)

EXAMPLES.

1. Reduce $\frac{1}{2}$ to fourths.

I can change the denominator to 4 by multiplying it by 2. But if I multiply the denominator, I must also multiply the numerator. Thus,

Process: $\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{2}{4}$, result.

2. Reduce $\frac{2}{3}$ to ninths.

Process: $\frac{2}{3}=\frac{3\times2}{3\times3}=\frac{6}{9}$, result.

How did I know to multiply by 3? Because the denominator *3* is to be changed to 9, and $3\times3=9$.

NOTE.—An integer may be treated as a fraction, having 1 for a denominator.

3. Reduce 7 to 5ths.

Process: $\frac{7}{1}=\frac{5\times7}{5\times1}=\frac{35}{5}$, result.

4. Reduce $\frac{5}{7}$ to 63ds.

Process: (1) $63\div7=9$.

(2) $\frac{5}{7}=\frac{9\times5}{9\times7}=\frac{45}{63}$, result.

Why did we divide 63 by 7?

5. Reduce $\frac{7}{13}$ to 195ths.

Process:

(1) $13\overline{)195}$ (15, 13, 65, 65)

(2) $\frac{7}{13}=\frac{15\times7}{15\times13}=\frac{105}{195}$, result.

EXERCISE LXXIII.

1. Reduce $\frac{1}{2}$ to 8ths.

2. Reduce $\frac{1}{7}$ to 28ths.

3. Reduce $\frac{5}{9}$ to 63ds.

4. Reduce $1\frac{2}{7}$ to 85ths.

5. Reduce $\frac{13}{19}$ to 190ths.
6. Reduce 9 to 24ths.
7. Reduce $\frac{11}{13}$ to 78ths.
8. Reduce $\frac{8}{21}$ to 252ds.
9. Reduce $\frac{1}{16}$ to 272ds.
10. Reduce $\frac{5}{11}$ to 3443ds.
11. Reduce 12 to 25ths.

82. Reducing Fractions to Lowest Terms.—A fraction is in its *Lowest Terms* when no number, except 1, is a factor of both numerator and denominator. *If both terms of a fraction be divided by the same number, the value of the fraction will not be changed.* (Commit.)

EXAMPLES.

1. Reduce $\frac{4}{6}$ to thirds.

Process: $\frac{4}{6}=\frac{4\div2}{6\div2}=\frac{2}{3}$, result.

How do we know to divide by 2? How many 3's in 6?

2. Reduce $\frac{12}{18}$ to lowest terms.

Divide by G. C. D. of both terms. The G. C. D. of 12 and 18 is 6.

Process: $\frac{12}{18}=\frac{12\div6}{18\div6}=\frac{2}{3}$, result; or $\frac{12}{18}=\frac{6)12}{6)16}=\frac{2}{3}$, result.

3. Reduce $\frac{49}{161}$ to lowest terms.

G. C. D. of 49 and 161 = 7.

Process: $\frac{49}{161}=\frac{49\div7}{161\div7}=\frac{7}{23}$, result; or $\frac{49}{161}=\frac{7)49}{7)161}=\frac{7}{23}$, result.

EXERCISE LXXIV.

Reduce to lowest terms:

1. $\frac{3}{9}$
2. $\frac{10}{12}$
3. $\frac{10}{15}$
4. $\frac{12}{16}$
5. $\frac{24}{48}$
6. $\frac{24}{60}$
7. $\frac{18}{72}$
8. $\frac{9}{90}$
9. $\frac{27}{63}$
10. $\frac{15}{60}$
11. $\frac{36}{100}$
12. $\frac{60}{108}$
13. $\frac{14}{56}$
14. $\frac{121}{143}$
15. $\frac{180}{441}$
16. $\frac{54}{144}$
17. $\frac{200}{325}$
18. $\frac{405}{1800}$
19. $\frac{540}{810}$
20. $\frac{1000}{1200}$
21. $\frac{999}{1998}$

83. Reducing Mixed Numbers to Improper Fractions.

EXAMPLES.

1. Reduce $3\frac{1}{2}$ to halves.

Process: (1) $3=\frac{2\times3}{2\times1}=\frac{6}{2}$.

(2) $\frac{6}{2}+\frac{1}{2}=\frac{7}{2}$, result.

(1) Reduce the integer to halves, (2) add the $\frac{1}{2}$.

Shorter Form: $3\frac{1}{2}=\frac{2\times3}{2}+\frac{1}{2}=\frac{7}{2}$, result.

2. $7\frac{3}{4}$ to 4ths.

Process: $7\frac{3}{4}=\frac{4\times7}{4}+\frac{3}{4}=\frac{31}{4}$, result.

3. Reduce $17\frac{9}{23}$ to an improper fraction.

Mechanical work:

17
23
51
34
391
9
400

Result $\frac{400}{23}$.

Explanation: 17 reduced to twenty-thirds will make 23×17, or 391 twenty-thirds. Adding the 9 twenty-thirds, we have 400 twenty-thirds, or $\frac{400}{23}$.

EXERCISE LXXV.

Reduce to improper fractions:

1. $3\frac{1}{8}$	*8.* $10\frac{1}{21}$	*15.* $21\frac{5}{46}$
2. $5\frac{1}{6}$	*9.* $14\frac{3}{7}$	*16.* $2\frac{1}{3}\frac{2}{4}\frac{1}{2}$
3. $6\frac{3}{11}$	*10.* $12\frac{3}{14}$	*17.* $17\frac{1}{11}\frac{1}{1}$
4. $7\frac{5}{9}$	*11.* $15\frac{5}{19}$	*18.* $10\frac{1}{100}$
5. $8\frac{8}{13}$	*12.* $17\frac{1}{17}$	*19.* $121\frac{3}{13}$
6. $21\frac{3}{4}$	*13.* $81\frac{1}{9}$	*20.* $846\frac{5}{164}$
7. $16\frac{2}{3}$	*14.* $73\frac{3}{11}$	*21.* $1\frac{999}{10000}$

84. Reducing Improper Fractions to Integers or Mixed Numbers.—Review Article 66.

EXAMPLES.

1. Reduce $\frac{20}{4}$ to an integer.

4)20
5, result.

NOTE.—As an expression of division, "*20*" is the dividend and "*4*" the divisor.

2. Reduce $\frac{355}{18}$ to a mixed number.

$19\frac{13}{18}$, result.

18)355
18
175
162
13

EXERCISE LXXVI.

Reduce to an integer or mixed number:

1. $\frac{8}{2}$
2. $\frac{24}{6}$
3. $\frac{17}{8}$
4. $\frac{45}{8}$
5. $\frac{75}{11}$
6. $\frac{124}{9}$
7. $\frac{246}{17}$
8. $\frac{1576}{9}$
9 $\frac{300}{13}$
10. $\frac{999}{19}$
11. $\frac{1000}{125}$
12. $\frac{11111}{111}$
13. $\frac{34687}{27}$
14. $\frac{8888}{81}$
15. $\frac{54832}{123}$

85. Reducing Fractions to Least Common Denominator.—The **Least Common Denominator** of two or more fractions is the L. C. M. of their denominators. The process of reduction is the same as in Article 81.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ to L. C. D.

The L. C. D. is 12. Then, the object is to reduce the several fractions to 12ths.

Process: (1) $\frac{1}{2}=\frac{6\times1}{6\times2}=\frac{6}{12}$

(2) $\frac{3}{4}=\frac{3\times3}{3\times4}=\frac{9}{12}$

(3) $\frac{5}{6}=\frac{2\times5}{2\times6}=\frac{10}{12}$

NOTE.—If necessary, review L. C. M. (See Article 76.)

2. Reduce $\frac{7}{12}$, $\frac{5}{18}$, $\frac{11}{24}$ to L. C. D.

Process: $\frac{7}{12}$, $\frac{6}{12}$, $\frac{11}{12}$.

(1) The L. C. D. is 72.

(2) $\frac{42}{72}$, $\frac{30}{72}$, $\frac{33}{72}$, result.

(3) $12)\overline{72}$; $\frac{6}{}$; $7\times6=42$.

(4) $18)\overline{72}$; $\frac{4}{}$; $5\times4=20$.

(5) $24)\overline{72}$; $\frac{3}{}$; $11\times3=33$.

NOTE.—The part on the right, which is the process of finding the numerators, may be put in such form as is convenient. As soon as the L. C. D. is found, the denominators may be placed in (2) thus: $\frac{}{72}$, $\frac{}{72}$, $\frac{}{72}$. Then, find the numerators and place them above.

EXERCISE LXXVII.

Reduce to Least Common Denominator:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
2. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$.
3. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$.
4. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{4}{9}$.
5. $\frac{6}{7}$, $\frac{8}{15}$, $\frac{5}{21}$.
6. $\frac{11}{21}$, $\frac{9}{35}$, $\frac{13}{27}$.
7. $\frac{11}{18}$, $\frac{13}{36}$, $\frac{5}{9}$.
8. $\frac{8}{21}$, $\frac{8}{35}$, $\frac{5}{42}$.
9. $\frac{3}{25}$, $\frac{33}{100}$, $\frac{7}{150}$.
10. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.

86. Addition of Fractions.—Reduce all the fractions to *Least Common Denominator* before adding.

EXAMPLES.

1. Add $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{6}$.

Process: $\frac{2}{3}+\frac{4}{5}+\frac{1}{6}=\frac{20}{30}+\frac{24}{30}+\frac{5}{30}=\frac{49}{30}=1\frac{19}{30}$, result.

Explanation: Mechanical work necessary to reduce the fractions to L. C. D. need not be preserved. 20 thirtieths + 24 thirtieths + 5 thirtieths = 49 thirtieths, or $\frac{49}{30}$. Just add the numerators and place the sum over the L. C. D.

NOTE.—If the result is an improper fraction, reduce it to an integer or mixed number.

2. Add $\frac{3}{4}$, $\frac{5}{9}$, $\frac{11}{12}$.

Process: $\frac{3}{4}+\frac{5}{9}+\frac{11}{12}=\frac{27}{36}+\frac{20}{36}+\frac{33}{36}=\frac{80}{36}=2\frac{8}{36}=2\frac{2}{9}$, result.

NOTE.—The $\frac{8}{36}$ should be reduced to its lowest terms, $\frac{2}{9}$. *In results, all fractions should be in their simplest forms.*

3. Add $6\frac{1}{8}$, $10\frac{4}{5}$, $5\frac{4}{15}$.

Process: (1) $\frac{1}{8}+\frac{4}{5}+\frac{4}{15}=$
$\frac{5}{15}+\frac{12}{15}+\frac{4}{15}=$
$\frac{21}{15}=1\frac{6}{15}=1\frac{2}{5}$

(2) $6\frac{1}{8}$
$10\frac{4}{5}$
$5\frac{4}{15}$
$22\frac{2}{5}$, result.

Explanation: Adding the fractions, we get $1\frac{2}{5}$. Write the $\frac{2}{5}$ below the fractions, carry the 1 and add it to the integers; result, $22\frac{2}{5}$.

EXERCISE LXXVIII.

Find the value of—

1. $\frac{3}{4}+\frac{5}{6}$
2. $\frac{3}{8}+\frac{7}{10}$
3. $\frac{5}{9}+\frac{8}{15}$
4. $1\frac{3}{5}+\frac{3}{25}$
5. $1\frac{7}{18}+1\frac{9}{27}$
6. $1\frac{4}{17}+\frac{2}{3}$
7. $\frac{5}{19}+\frac{7}{13}$
8. $\frac{8}{15}+1\frac{2}{25}$
9. $\frac{1}{20}+\frac{1}{30}$
10. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$
11. $\frac{2}{5}+\frac{5}{8}+\frac{3}{10}$
12. $\frac{3}{9}+\frac{7}{15}+\frac{4}{21}$
13. $\frac{8}{15}+\frac{5}{6}+\frac{7}{25}$
14. $\frac{8}{33}+\frac{7}{44}+1\frac{3}{22}$
15. $7\frac{1}{2}+8\frac{1}{3}$
16. $12\frac{3}{5}+14\frac{3}{8}$
17. $21\frac{3}{10}+30\frac{5}{12}$
18. $1\frac{1}{8}+14\frac{3}{7}$
19. $5\frac{3}{17}+8\frac{6}{17}$
20. $\frac{3}{8}+5\frac{1}{9}+6$
21. $3\frac{1}{2}+4\frac{1}{4}+5\frac{1}{5}$
22. $17+2\frac{1}{10}+7\frac{6}{7}$
23. $6\frac{1}{4}+\frac{5}{9}+\frac{7}{12}$
24. $17\frac{5}{13}+17\frac{3}{9}+17\frac{1}{3}$

25. Jane had $\$\frac{1}{4}$ and her brother $\$\frac{1}{5}$. How much money did both have?

Question: $\$\frac{1}{4}+\$\frac{1}{5}=\$(\ )$? *Mechanical Work:*
Answer: $\$\frac{1}{4}+\$\frac{1}{5}=\$\frac{9}{20}$. $\frac{1}{4}+\frac{1}{5}=\frac{5}{20}+\frac{4}{20}=\frac{9}{20}$.

NOTE.—The *question* should be written *first;* then there is no doubt about what the problem calls for. *Mechanical work* may be omitted or preserved, at the discretion of the teacher.

26. One boy ate $\frac{1}{4}$ of a melon, another ate $\frac{1}{5}$ of it, and a third $\frac{1}{6}$ of it. What part of the melon did all eat?

27. $\frac{1}{5}$ of my money is in dollars, and $\frac{3}{4}$ of it in fifty-cent pieces. What part of my money is in these coins?

28. I sold $\frac{5}{8}$ of my crop to Mr. Brown and $\frac{1}{6}$ of it to Mr. Jones. What part of the crop have I sold?

Question: $\frac{5}{8}$ of crop $+\frac{1}{6}$ of crop $=(\ )$ of crop?

29. I pay $\frac{2}{9}$ of my salary for board and $\frac{3}{10}$ of it for clothing. What part of my salary do I pay for food and clothes?

30. My father gave me $8\frac{3}{10}$, my mother gave me $5\frac{9}{20}$. How much money did I then have?

31. From a bin of wheat, I sold to one man $30\frac{3}{4}$ bu., to another $27\frac{1}{2}$ bu., and to a third $17\frac{1}{8}$ bu. How many bushels did I sell?

32. There are $5\frac{1}{2}$ gallons of milk in one can, $2\frac{5}{8}$ gallons in another, and $3\frac{3}{4}$ gallons in another. How many gallons in the three cans?

33. On one Saturday, Mr. Williams deposited in the bank $\$72\frac{4}{5}$; on the next Saturday, he deposited $\$103\frac{3}{10}$; and on the third Saturday, he deposited $\$98\frac{3}{4}$. How much money did he deposit in the three Saturdays?

34. James has $3\frac{5}{12}$ dozen apples; John, $6\frac{5}{6}$ dozen; and Charles, $3\frac{2}{3}$ dozen. How many dozen in all?

35. I bought two loads of corn: one contained $20\frac{3}{4}$ bushels, and cost $\$5\frac{3}{16}$; the other contained $17\frac{5}{8}$ bushels, and cost $\$5\frac{4}{15}$. How many bushels did I buy, and how much did it cost?

87. Subtraction of Fractions.—Reduce the fractions to *Least Common Denominator* before subtracting.

EXAMPLES.

1. From $\frac{5}{9}$ take $\frac{5}{12}$.

Process: $\frac{5}{9} - \frac{5}{12} = \frac{20}{36} - \frac{15}{36} = \frac{5}{36}$, result.

Explanation: 20 thirty-sixths − 15 thirty-sixths = 5 thirty-sixths, or $\frac{5}{36}$. After reducing to L. C. D., subtract the numerator of the subtrahend from the numerator of the minuend, and place the remainder over the L. C. D. *In the result, always reduce the fraction to its simplest form.*

2. From $17\frac{1}{3}$ take $14\frac{1}{4}$.

Process: (1) $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$.

(2) $17\frac{1}{3}$
$14\frac{1}{4}$
$3\frac{1}{12}$, result.

3. From $17\frac{1}{7}$ take $5\frac{8}{9}$.

Process: (1) $\frac{9}{9}-\frac{8}{9}=\frac{72}{63}-\frac{56}{63}=\frac{16}{63}$.

(2) $17\frac{1}{7}$
$5\frac{8}{9}$
$11\frac{16}{63}$, result.

Explanation: Since $\frac{8}{9}$ is larger than $\frac{1}{7}$, 1 must be taken from 17 and added to the $\frac{1}{7}$, making $1\frac{1}{7}$ or $\frac{8}{7}$; $\frac{8}{7}-\frac{8}{9}=\frac{16}{63}$. This leaves only 16 in the minuend, from which to subtract the 5.

EXERCISE LXXIX.

Find the value of—

1. $\frac{3}{4}-\frac{3}{8}$	*11.* $\frac{1}{15}-\frac{1}{25}$	*21.* $50\frac{3}{16}-18\frac{5}{9}$
2. $\frac{1}{4}-\frac{1}{6}$	*12.* $7\frac{1}{3}-\frac{1}{7}$	*22.* $44\frac{4}{7}-5\frac{8}{15}$
3. $\frac{5}{9}-\frac{1}{10}$	*13.* $6\frac{4}{13}-\frac{2}{15}$	*23.* $122\frac{1}{19}-6\frac{3}{38}$
4. $\frac{7}{12}-\frac{3}{10}$	*14.* $8-\frac{3}{5}$	*24.* $37\frac{1}{2}-\frac{1}{27}$
5. $\frac{8}{15}-\frac{1}{4}$	*15.* $11\frac{7}{10}-1\frac{5}{16}$	*25.* $8\frac{3}{11}-6\frac{1}{15}$
6. $\frac{3}{9}-\frac{7}{16}$	*16.* $33\frac{1}{3}-16\frac{2}{3}$	*26.* $13\frac{3}{10}-4\frac{4}{5}$
7. $\frac{1}{12}-\frac{1}{18}$	*17.* $18\frac{1}{2}-10\frac{3}{10}$	*27.* $29\frac{1}{7}-18\frac{3}{4}$
8. $\frac{5}{18}-\frac{5}{21}$	*18.* $16\frac{2}{3}-11\frac{5}{12}$	*28.* $17\frac{1}{8}-12\frac{1}{6}$
9. $\frac{1}{14}-\frac{1}{21}$	*19.* $120-14\frac{3}{7}$	*29.* $14\frac{3}{13}-8\frac{2}{7}$
10. $\frac{3}{11}-\frac{5}{22}$	*20.* $84\frac{4}{9}-18\frac{8}{9}$	*30.* $24\frac{1}{18}-23\frac{1}{19}\frac{8}{9}$

31. I had $\$2\frac{3}{10}$, and gave $\$\frac{3}{4}$ for a book. How much money had I left?

Question: $\$2\frac{3}{10}-\$\frac{3}{4}=\$(\ )$?
Answer: $\$2\frac{3}{10}-\$\frac{3}{4}=\$1\frac{11}{20}$.

Mechanical Work:
$2\frac{3}{10}-\frac{3}{4}=\frac{23}{10}-\frac{3}{4}=$
$\frac{46}{20}-\frac{15}{20}=\frac{31}{20}=1\frac{11}{20}$.

NOTE.—*Mechanical work* may be preserved or omitted, at the discretion of the teacher.

32. I own a mill, and sell $\frac{5}{17}$ of it. What part have I left?

33. I have $\$17\frac{3}{25}$, but owe Mr. Smith $\$14\frac{7}{10}$. How much will I have left after paying the debt?

34. There are $44\frac{5}{12}$ yards in a certain bolt of cloth. After the merchant sells $15\frac{4}{9}$ yards, how much is left?

35. A vessel contained $\frac{7}{12}$ of a gallon of milk; from it was poured $\frac{1}{8}$ of a gallon. How much was left?

36. A book cost $\$\frac{3}{5}$, a scratch-book cost $\$\frac{3}{10}$. How much more did the book cost than the scratch-book?

37. What sum added to $\$1.43\frac{3}{4}$ will make $\$2.16\frac{5}{12}$?

38. A boy starts to market with $3\frac{3}{4}$ dozen of eggs; but after breaking some, he finds that he has only $1\frac{5}{6}$ dozen. How many dozen did he break?

88. Multiplication of Fractions.—When the multiplier is a fraction, it may be followed by the word "*of*" or the sign, "×." Thus,

$$\frac{3}{5} \text{ of } 10 = \frac{3}{5} \times 10.$$

Reduce *mixed numbers* to *improper fractions* before multiplying.

EXAMPLES.

1. Find $\frac{3}{7}$ of 35.

Process Complete: (1) $\frac{1}{7}$ of $35 = \frac{35}{7} = 5$. (See Art. 64, 3d Ap.)
$3 \times (1) = (2)$ $\frac{3}{7}$ of $35 = 3 \times 5 = 15$, result.

Process Shortened: $\frac{3 \times \overset{5}{\cancel{35}}}{\underset{1}{\cancel{7}}} = 15$, result.

2. Find $\frac{1}{7}$ of $\frac{2}{3}$.

Process: $\frac{1}{7}$ of $\frac{2}{3} = \frac{2}{3} \div 7 = \frac{2}{21}$, result.

Explanation: $\frac{2}{3}$ is $2 \div 3$. Dividing this by 7 is the same as dividing 2 by 3×7, or 21. $2 \div 21 = \frac{2}{21}$.

3. Find $\frac{3}{7}$ of $\frac{2}{3}$.

Process Complete: (1) $\frac{1}{7}$ of $\frac{2}{3} = \frac{2}{21}$.
$3 \times (1) = (2)$ $\frac{3}{7}$ of $\frac{2}{3} = 3 \times \frac{2}{21} = \frac{6}{21} = \frac{2}{7}$, result.

Process Shortened: $\frac{3}{7}$ of $\frac{2}{3} = \frac{\overset{1}{\cancel{3}} \times 2}{7 \times \underset{1}{\cancel{3}}} = \frac{2}{7}$, result.

NOTE.—By the *short process*, we express all the numbers as fractions, then multiply all numerators together for a new numerator, and all denominators together for a new denominator. *Employ cancellation.*

4. Find $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$.

Process: $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9} = \frac{1 \times \overset{1}{\cancel{3}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{4}} \times \underset{3}{\cancel{9}}} = \frac{1}{3}$, result.

5. $7\frac{1}{2} \times \frac{3}{25} = (\quad)$?

Process: $7\frac{1}{2} \times \frac{3}{25} = \frac{\overset{3}{\cancel{15}} \times 3}{2 \times \underset{5}{\cancel{25}}} = \frac{9}{10}$, result.

6. Multiply $\frac{3}{4}$ of 8 by $\frac{1}{3}$ of 6.

Process: $\frac{3}{4}$ of $8 \times \frac{1}{3}$ of $6 = \frac{\overset{2}{\cancel{3}} \times \cancel{8} \times 1 \times \cancel{6}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}}} = 12$, result.

EXERCISE LXXX.

Find the value of—

1. $\frac{1}{3}$ of $\frac{3}{4}$
2. $2 \times \frac{1}{4}$
3. $\frac{4}{5} \times 5$
4. $\frac{7}{8} \times 12$
5. $9 \times \frac{3}{4}$
6. $10 \times \frac{3}{5}$
7. $\frac{1}{6}$ of $5\frac{4}{5}$
8. $\frac{3}{7}$ of $4\frac{1}{5}$
9. $5\frac{1}{8} \times 4$
10. $7\frac{1}{7} \times \frac{7}{20}$
11. $\frac{5}{9}$ of $7\frac{1}{5}$
12. $8\frac{3}{11} \times \frac{8}{24}$
13. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$
14. $\frac{5}{12}$ of $\frac{8}{12}$ of $4\frac{2}{5}$
15. $8 \times \frac{3}{10}$ of $\frac{5}{6}$
16. $7\frac{1}{2} \times 8\frac{4}{15} \times 1\frac{1}{4}$
17. $\frac{5}{9}$ of $\frac{3}{24}$ of $\frac{8}{25}$
18. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$
19. $\frac{5}{12}$ of $\frac{6}{10}$ of $\frac{8}{15}$ of $\frac{1}{10}$
20. $3\frac{1}{4} \times \frac{3}{26} \times \frac{7}{10} \times 5\frac{1}{4}$
21. $8\frac{3}{8} \times \frac{5}{9} \times \frac{3}{7} \times 1\frac{1}{2}$
22. $\frac{3}{13}$ of $\frac{26}{9} \times \frac{8}{25} \times 6\frac{1}{4}$
23. $5 \times 6\frac{1}{6} \times \frac{11}{37} \times 8\frac{2}{11}$
24. $\frac{5}{12} \times \frac{6}{13} \times \frac{7}{14} \times \frac{8}{24} \times 64$.

89. Problems.

EXAMPLES.

1. If a ton of hay is worth $\$4\frac{1}{4}$, what is $\frac{3}{5}$ of a ton worth?

Solution: (1) Price of 1 T. = $\$4\frac{1}{4}$. (Basis.)
$\frac{3}{5}$ of (1) = (2) Price of $\frac{3}{5}$ T. = $\$\frac{51}{10} = \$2\frac{1}{10}$, result.

2. What will $\frac{3}{4}$ bushel of wheat cost at $\$\frac{2}{3}$ per bushel?

Solution: (1) Cost of 1 bu. = $\$\frac{2}{3}$. (Basis.)
$\frac{3}{4}$ of (1) = (2) Cost of $\frac{3}{4}$ bu. = $\$\frac{1}{2}$, result.

NOTE.—In preparing lesson or putting work on blackboard, mechanical work need not appear with the solution; yet the teacher should be exacting about accuracy of results.

3. What will 25 bushels of corn bring at $\$\frac{3}{8}$ per bushel?

Solution: (1) Cost of 1 bu. $= \$\frac{3}{8}$. (Basis.)
$25 \times (1) = (2)$ Cost of 25 bu. $= \$9\frac{3}{8}$, result.

4. If 6 oranges cost $30\cent$, what will 1 orange cost?

Solution: (1) Cost of 6 oranges $= 30\cent$. (Basis.)
$\frac{1}{6}$ of $(1) = (2)$ Cost of 1 orange $= 5\cent$, result.

5. Reduce $\frac{1}{2}$ bu. $\frac{3}{4}$ pk. $\frac{1}{2}$ qt. to pints.

Solution: (1) 1 bu. $= 4$ pk. (Basis.)
$\frac{1}{2}$ of $(1) = (2)$ $\frac{1}{2}$ bu. $= 2$ pk.
(3) 2 pk. $+ \frac{3}{4}$ pk. $= 2\frac{3}{4}$ pk.
(4) 1 pk. $= 8$ qt. (Basis.)
$2\frac{3}{4} \times (4) = (5)$ $2\frac{3}{4}$ pk. $= 22$ qt.
(6) 22 qt. $+ \frac{1}{2}$ qt. $= 22\frac{1}{2}$ qt.
(7) 1 qt. $= 2$ pt. (Basis.)
$22\frac{1}{2} \times (7) = (8)$ $22\frac{1}{2}$ qt. $= 45$ pt., result.

6. Reduce $\frac{1}{5}$ yr. $\frac{1}{5}$ da. $\frac{1}{5}$ hr. $\frac{1}{5}$ min. to seconds.

Solution: (1) 1 yr. $= 12$ mo. (Basis.)
$\frac{1}{5}$ of $(1) = (2)$ $\frac{1}{5}$ yr. $= 2\frac{2}{5}$ mo.
(3) 1 mo. $= 30$ da. (Basis.)
$2\frac{2}{5} \times (3) = (4)$ $2\frac{2}{5}$ mo. $= 72$ da.
(5) 72 da. $+ \frac{1}{5}$ da. $= 72\frac{1}{5}$ da.
(6) 1 da. $= 24$ hr. (Basis.)
$72\frac{1}{5} \times (6) = (7)$ $72\frac{1}{5}$ da. $= 1732\frac{4}{5}$ hr.
(8) $1732\frac{4}{5}$ hr. $+ \frac{1}{5}$ hr. $= 1733$ hr.
(9) 1 hr. $= 60$ min. (Basis.)
$1733 \times (9) = (10)$ 1733 hr. $= 103980$ min.
(11) 103980 min. $+ \frac{1}{5}$ min. $= 103980\frac{1}{5}$ min.
(12) 1 min. $= 60$ sec. (Basis.)
$103980\frac{1}{5} \times (12) = (13)$ $103980\frac{1}{5}$ min. $= 6238812$ sec., result.

7. Reduce 6 inches to the fraction of a yard.

Solution: (1) 1 in. $= \frac{1}{12}$ ft. (Basis.)
$6 \times (1) = (2)$ 6 in. $= \frac{1}{2}$ ft.
(3) 1 ft. $= \frac{1}{3}$ yd. (Basis.)
$\frac{1}{2}$ of $(3) = (4)$ $\frac{1}{2}$ ft. $= \frac{1}{6}$ yd., result.

EXERCISE LXXXI.

1. A man bought $12\frac{1}{2}$ pounds of tea, at $\$\frac{4}{5}$ per pound. How much did it cost him?

2. How much will $7\frac{1}{4}$ bushels of potatoes cost, at $\$\frac{7}{8}$ per bushel?

3. If 25 men cut $33\frac{1}{3}$ acres of corn, how much does each cut?

4. If 7 books cost $\$4\frac{2}{3}$, what cost 1 book?

5. Find the cost of $15\frac{3}{4}$ dozen eggs, at $9\frac{1}{3}¢$ per dozen.

6. I sold a chicken which weighed $6\frac{1}{4}$ pounds, at $8\frac{1}{3}¢$ per pound. How much did I get for it?

7. What will $3\frac{5}{16}$ pounds of meat cost, at $10¢$ per pound?

8. A man can travel $25\frac{1}{2}$ miles per day. How far can he travel in $4\frac{2}{7}$ days?

9. What will $\frac{3}{7}$ of a cord of wood cost, at $\$1\frac{1}{4}$ per cord?

10. If 9 men earn $\$22\frac{1}{2}$, what will 1 man earn?

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11. Reduce $\frac{1}{2}$ pound to ounces.

12. Reduce $3\frac{1}{4}$ yards $\frac{1}{2}$ foot to inches.

13. Reduce $\frac{1}{2}$ yard $\frac{3}{4}$ foot to inches.

14. Reduce $7\frac{3}{5}$ tons $\frac{3}{16}$ hundredweight $12\frac{3}{8}$ pounds to ounces.

15. What is the value of $3\frac{3}{4}$ gallons $2\frac{1}{2}$ quarts of milk at $2\frac{1}{2}¢$ per pint?

16. What is the cost of $3\frac{1}{2}$ bushels $2\frac{1}{2}$ pecks of plums at $6¢$ per quart?

17. Reduce 1 foot 6 inches to the fraction of a yard.

18. Reduce $2\frac{1}{4}$ pints to the fraction of a gallon.

90. Division of Fractions.—*The easiest way to divide in fractions is to invert the divisor and proceed as in multiplication of fractions.* To explain, we know that—

1 apple contains $\frac{1}{2}$ apple 2 times.
1 apple contains $\frac{1}{3}$ apple 3 times.
1 apple contains $\frac{1}{4}$ apple 4 times.
1 apple contains $\frac{2}{4}$ apple 2 times.
1 apple contains $\frac{1}{6}$ apple 6 times.
1 apple contains $\frac{2}{6}$ apple 3 times, etc.

Now, let us arrange these expressions in an abstract form:

$1 \div \frac{1}{2} = \frac{2}{1}$, or 2.
$1 \div \frac{1}{3} = \frac{3}{1}$, or 3.
$1 \div \frac{1}{4} = \frac{4}{1}$, or 4.
$1 \div \frac{2}{4} = \frac{4}{2}$, or 2.
$1 \div \frac{1}{6} = \frac{6}{1}$, or 6.
$1 \div \frac{2}{6} = \frac{6}{2}$, or 3.

Look at each divisor and the first form of the corresponding quotient, and you will see that, in each case, the *quotient* is the *divisor inverted.* This is an illustration of the *general truth: Inverting a divisor shows how many times that divisor is contained in one. Commit this to memory.* With this *truth* in mind, let us study the following examples:

EXAMPLES.

1. Divide 14 by $\frac{7}{8}$.

Process Complete: (1) $1 \div \frac{7}{8} = \frac{8}{7}$. (Above truth.)

$14 \times (1) = (2)$ $14 \div \frac{7}{8} = \overset{2}{\cancel{14}} \times \frac{8}{\cancel{7}} = 16$, result.

Process Shortened: $14 \div \frac{7}{8} = \overset{2}{\cancel{14}} \times \frac{8}{\cancel{7}} = 16$, result.

2. $\frac{3}{4} \div \frac{7}{8} = (\quad)$?

Process Complete: (1) $1 \div \frac{7}{8} = \frac{8}{7}$.

$\frac{3}{4}$ of $(1) = (2)$ $\frac{3}{4} \div \frac{7}{8} = \frac{3}{\cancel{4}}$ of $\frac{\overset{2}{\cancel{8}}}{7} = \frac{6}{7}$, result.

Process Shortened: $\frac{3}{4} \div \frac{7}{8} = \frac{3}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{7} = \frac{6}{7}$, result.

3. $1\frac{2}{17} \div 8 = (\quad)$?

Process Complete: (1) $1 \div 8 = \frac{1}{8}$.

$\frac{12}{17}$ of $(1) = (2)$ $\frac{12}{17} \div 8 = \frac{\overset{3}{\cancel{12}}}{17}$ of $\frac{1}{\cancel{8}_{2}} = \frac{3}{34}$, result.

Process Shortened: $\frac{12}{17} \div 8 = \frac{\overset{3}{\cancel{12}}}{17} \times \frac{1}{\underset{2}{\cancel{8}}} = \frac{3}{34}$, result.

NOTE.—The first step in the complete process gives us the divisor inverted; this may be done *mentally*. The "*Shortened Process*" is simply the last step in the *Complete Process*, and may be expressed thus: *Invert the divisor and proceed as in multiplication of fractions.* Use the *shorter process* in practice.

4. Divide $7\frac{1}{2}$ by $3\frac{1}{3}$.

Process: $7\frac{1}{2} \div 3\frac{1}{3} = \frac{\overset{3}{\cancel{15}}}{2} \times \frac{3}{\underset{2}{\cancel{10}}} = \frac{9}{4}$, or $2\frac{1}{4}$, result.

5. $\frac{12\frac{1}{5}}{1\frac{1}{2}} = (\quad)$?

NOTE.—$\frac{12\frac{1}{5}}{1\frac{1}{2}}$ is a complex fraction, and means, like any other fraction, that its numerator, $12\frac{1}{5}$, is to be divided by its denominator, $1\frac{1}{2}$. Then the process is easy:

Process: $\frac{12\frac{1}{5}}{1\frac{1}{2}} = 12\frac{1}{5} \div 1\frac{1}{2} = \frac{61}{5} \times \frac{2}{3} = \frac{122}{15}$, or $8\frac{2}{15}$, result.

EXERCISE LXXXII.

Find the value of—

1. $\frac{3}{5} \div 9$
2. $1\frac{1}{3} \div 16$
3. $5\frac{1}{2} \div 33$
4. $2\frac{2}{5} \div 18$
5. $6\frac{3}{7} \div 30$
6. $10 \div \frac{4}{7}$
7. $8 \div \frac{1}{4}$
8. $7 \div 3\frac{2}{3}$
9. $9 \div \frac{5}{6}$
10. $\frac{7}{10} \div \frac{5}{8}$
11. $\frac{9}{16} \div \frac{3}{4}$
12. $\frac{7}{11} \div 2\frac{5}{8}$
13. $\frac{5}{9} \div \frac{5}{8}$
14. $2\frac{3}{5} \div 2\frac{6}{45}$
15. $18\frac{2}{3} \div 6\frac{1}{4}$
16. $37\frac{1}{2} \div 8\frac{1}{3}$
17. $\frac{124\frac{3}{4}}{71\frac{2}{7}}$
18. $\frac{241\frac{2}{13}}{29\frac{5}{11}}$
19. $\frac{8\frac{8}{8}}{6\frac{7}{10}}$
20. $\frac{341}{20\frac{1}{17}}$

91. Problems.

EXAMPLES.

1. If three parcels cost $\$1\frac{3}{16}$, how many parcels can be bought for $\$\frac{7}{8}$?

Solution: (1) $\$\frac{7}{8}$ = cost of ( ) parcels? (Question.)
(2) $\$1\frac{3}{16}$ = cost of 3 parcels. (Basis.)
(3) $\frac{7}{8} : 1\frac{3}{16} :: (\ ) : 3$? (Proportion.)
(4) $\frac{7 \times \cancel{3} \times \overset{2}{\cancel{16}}}{\cancel{8} \times \cancel{3}} = 14.$
∴ the required answer is 14 parcels.

NOTE.—Why is the 16 above and the 3 below in (4)? Because $1\frac{3}{16}$ becomes the divisor and is *inverted.*

2. If $\frac{1}{6}$ of a bolt of muslin costs $\$\frac{2}{3}$, what will $\frac{8}{9}$ of a bolt cost?

Solution: (1) Cost of $\frac{8}{9}$ bolt = $\$(\ )$? (Question.)
(2) Cost of $\frac{1}{6}$ bolt = $\$\frac{2}{3}$. (Basis.)
(3) $\frac{8}{9} : \frac{1}{6} :: (\ ) : \frac{2}{3}$? (Proportion.)
(4) $\frac{8 \times 2 \times \overset{2}{\cancel{6}}}{9 \times \cancel{3} \times 1} = \frac{32}{9} = 3\frac{5}{9}.$
∴ the required answer is $\$3\frac{5}{9}$.

3. If $3\frac{1}{2}$ tons of coal cost $\$9\frac{5}{8}$, what will $7\frac{1}{5}$ tons cost?

Solution: (1) Cost of $7\frac{1}{5}$ tons = $\$(\ )$? (Question.)
(2) Cost of $3\frac{1}{2}$ tons = $\$9\frac{5}{8}$. (Basis.)
(3) $7\frac{1}{5} : 3\frac{1}{2} :: (\ ) : 9\frac{5}{8}$? (Proportion.)
(4) $\frac{\overset{9}{\cancel{36}} \times \overset{11}{\cancel{77}} \times \overset{2}{\cancel{2}}}{5 \times \underset{4}{\cancel{8}} \times \cancel{7}} = \frac{99}{5} = 19\frac{4}{5}.$
∴ the required answer is $\$19\frac{4}{5}$.

EXERCISE LXXXIII.

1. How many bushels of apples at $\$\frac{3}{8}$ per bushel can I buy for $\$3\frac{3}{4}$?

2. A man divided \$6 among some beggars, giving each $\$\frac{3}{10}$. How many beggars were there?

3. A man puts 3648 bushels of grain into bags, each containing $2\frac{2}{3}$ bushels. How many bags were used?

4. An English pound (£) is worth $\$4\frac{7}{10}$; how many £'s are worth $\$596\frac{1}{10}$?

5. How many dozen eggs at $\$\frac{1}{8}$ per dozen can be bought for $\$4\frac{3}{32}$?

6. How many fields of $17\frac{1}{2}$ acres each may be made from $227\frac{1}{2}$ acres of land?

7. I spent $\$2\frac{1}{2}$ for books, paying $\$\frac{1}{6}$ for each. How many books did I buy?

8. I divided $\$18\frac{3}{4}$ among some boys, giving to each $\$3\frac{1}{8}$. How many boys were there?

9. If $5\frac{1}{2}$ yards make a rod, how many rods in $236\frac{1}{2}$ yards?

PART III.

I. ADDITION.

92. Review.

EXERCISE LXXXIV.

1. What is a *unit?* What is a *number?*

2. In reading and writing numbers, how many figures in a period? What are the *places* in a period called? Name the first three periods.

3. What is *addition?* What are *addends?* What is the *result* of addition called? What is the sign of addition called? Is the sign placed after or before the number to be added?

4. Can you add 5 *horses* and 6 *hats?* What *kind* of numbers may be added together?

5. When addends are large, how do you write them for addition? Why? Would it be possible to add them, if written otherwise?

6. How many units of one place make one of the next higher place? Explain "*carrying.*"

7. Do you usually add upward or downward? Can you add either way?

8. How many places are used for *cents*, when written with the "$"? What is always put just to the left of cents, when written in this way?

9. How many cents in one dollar? A cent is what part of a dollar?

10. How do you write numbers for addition, when the addends contain both *dollars* and *cents?*

NOTE.—Require *accuracy* and *speed* of the pupil in adding the 12 following points:

11.	*12.*	*13.*	*14.*
548	8005	35946	84362
6437	94897	2408	59462
2108	47651	13579	99999
29	3221	60204	273
6572	92876	59406	84063
4090	76543	88888	56789

15.	*16.*	*17.*	*18.*
23045	47682	33333	25
8467	5987	4376	473
10206	11111	59487	6759
56073	20720	27	87063
7777	120476	567	987564
85943	930784	88888	74689
5678	359067	90870	5964
94316	798632	67895	426
104627	987678	99999	274562

19.	*20.*	*21.*	*22.*
$264.75	$84.60	$8046.80	$1346.20
$849.50	$948.20	$7654.21	$5924.80
$5463.25	$7654.25	$9897.87	$247.20
$9076.00	$9876.50	$7642.51	$19.75
$4860.25	$6452.83	$9876.59	$6583.25
$606.64	$1074.60	$8384.85	$4444.40
$1728.50	$6666.66	$6733.33	$5896.40

23. January has 31 days; February, 28; March, 31; April, 30; May, 31; June, 30; July, 31; August, 31; September, 30; October, 31; November, 30; December, 31. How many days in the year?

24. How many times will a clock strike between 5 minutes after noon and 5 minutes after the next midnight?

25. A certain thresher threshed on Monday, 586 bushels of wheat; on Tuesday, 743 bushels; on Wednesday, 1060 bushels; on Thursday, 1264 bushels; on Friday, 920 bushels; and on Saturday, 840 bushels. How many bushels did it thresh in the week?

26. In a certain city school, the daily attendance for the month was as follows: In the first ward, 3275; in the second ward, 4863; in the third ward, 4989; in the fourth ward, 3674. What was the total daily attendance for the month?

27. Mr. Jones has 170 acres of land; Mr. Lewis has 540 acres more than Mr. Jones; and Mr. Gibbs has 570 acres more than Mr. Lewis. How many acres has Mr. Lewis? How many acres has Mr. Gibbs?

28. Mr. Brown's merchandise unsold, Aug. 29, 1900: Nails, $976; assorted gate hinges, $52.50; door locks, $180.76; wheelbarrows, $47.80; carpenters' tools, $75.95; plows, $524; garden tools, $65. Find the total value of the merchandise.

29. In addition to the merchandise in No. 28, Mr. Brown has other people's notes to the value of $2174; cash on hand, $1398.54; and personal accounts owing to him, $2954. How much is he worth in all?

30. Add together the successive integers from 1846 to 1854, including these numbers.

93. Addition of Compound Numbers.—A Simple Number *is a number whose integral units are of the same size and kind.* As, 250 books; 56 bushels.

All our study up to this time has been about simple numbers. **A Compound Number** is a number whose integral units are of two or more sizes, but of the same kind. As,

4 bu. 3 pk. 5 qt. 1 pt.; 5 yd. 2 ft. 6 in.

NOTE.—The word "kind" used above means kind as to nature of the units; if one unit in a compound number is a *linear unit*, all must be *linear units* of some size; if one unit is a unit of *dry measure*, all must be *dry-measure units*, etc.

EXAMPLES.

1. Add 1 bu. 2 pk. 4 qt. 1 pt.; 5 bu. 1 pk. 2 qt.

Process:

bu.	pk.	qt.	pt.	
1	2	4	1	
5	1	2	0	
6	3	6	1	result.

Explanation: (1) Write the names of the successive units of the table used, putting them in horizontal order, with the largest to the left.

(2) Write the addends below—each part in its place, as indicated by the table above.

(3) Add as we have done heretofore, placing the sum obtained by adding each column below. *Result, 6 bu. 3 pk. 6 qt. 1 pt.*

2. Add 2 gal. 3 qt. 1 pt. 3 gi.; 7 gal. 1 pt. 3 gi.; 8 gal. 2 qt. 3 gi.

Process:

gal.	qt.	pt.	gi.	
2	3	1	3	
7	0	1	3	
8	2	0	3	
18	3	0	1	result.

Explanation: (1) 3 gi. + 3 gi. + 3 gi. = 9 gi. But 9 gi. = 2 pt. 1 gi. Write the *1* below, and carry the *2 pt.*

(2) 2 pt. (carried) + 1 pt. + 1 pt. = 4 pt. But 4 pt. = 2 qt. Write *0* below, and carry the *2 qt.* We use the *0* to show that there are no pints left.

(3) 2 qt. (carried) + 2 qt. + 3 qt. = 7 qt. = 1 gal. 3 qt. Write the *3* below, and carry the *1 gal.*

(4) 1 gal. (carried) + 8 gal. + 7 gal. + 2 gal. = 18 gal. Write 18 below. *Result, 18 gal. 3 qt. 1 gi.*

NOTE.—Teacher should point out the similarity between this and *Addition of Simple Numbers.*

3. Add 5 yd. 2 ft. 8 in.; 7 yd. 2 ft. 9 in.; 17 yd. 2 ft. 11 in.

Process:

yd.	ft.	in.	
5	2	8	
7	2	9	
17	2	11	
31	2	4	result.

Explanation: (1) Sum, 28 in. = 2 ft. 4 in. Write the *4* below, and carry the *2.*

(2) Sum, 8 ft. = 2 yd. 2 ft. Write the *2* (ft.) below, and carry the *2* (yd.).

(3) Sum, 31 yd. Write the *31* below. *Result, 31 yd. 2 ft. 4 in.*

EXERCISE LXXXV.

Add:

1.

hr.	min.	sec.
5	15	45
7	35	30
6	20	15

2.

yr.	mo.	da.
1	8	15
5	9	20
3	6	18

3.

yd.	ft.	in.
7	1	10
5	2	8
15	1	9

4.

bu.	pk.	qt.	pt.
7	2	6	1
9	3	7	1
12	0	5	1

5.

gal.	qt.	pt.	gi.
11	3	0	3
9	2	1	2
18	3	1	3

6.

T.	cwt.	lb.	oz.
8	7	65	11
5	15	44	14
7	12	50	15

7. A farmer sold four loads of wheat: the first load contained 20 bu. 2 pk.; the second, 34 bu. 3 pk. 7 qt.; the third, 40 bu. 6 qt.; the fourth, 35 bu. 1 pk. 3 qt. Find the amount.

8. On Monday, a milkman sells 15 gal. 3 qt. 1 pt.; on Tuesday, 16 gal. 1 pt. 2 gi.; on Wednesday, 18 gal. 2 qt. 1 pt. 2 gi.; on Thursday, 12 gal. 3 qt. 1 pt. 2 gi.; on Friday, 18 gal. 3 qt.; on Saturday, 17 gal. 1 qt. 1 pt. 2 gi. How much milk does he sell in the six days?

9. A grocer bought 3 barrels of sugar: the first weighed 2 cwt. 40 lb.; the second, 2 cwt. 60 lb.; the third, 1 cwt. 96 lb. How much did all weigh?

10. I measured 4 bolts of cloth: the first measured 36 yd. 1 ft. 6 in.; the second, 52 yd. 2 ft.; the third, 48 yd. 2 ft. 4 in.; the fourth, 47 yd. 1 ft. 9 in. Find the sum of the measurements.

II. SUBTRACTION.

94. Review.

EXERCISE LXXXVI.

1. Define *Subtraction*, *Minuend*, *Subtrahend*, *Remainder*.

2. Can you subtract 5 hats from 6 books? What must be the kind of the *minuend* and *subtrahend* in subtraction? What is the sign of subtraction called? Is the sign placed after or before the number to be subtracted?

3. Which must be the larger, the minuend or subtrahend?

4. When the numbers are large, how do you write them for subtraction?

5. Using 506 as a minuend and 374 as subtrahend, explain fully the process of subtraction.

NOTE.—Require *accuracy* and *speed* of the pupil in the following subtractions:

6.	*7.*	*8.*	*9.*
56703	30876	77777	55555
48693	16897	67890	34567

10.	*11.*	*12.*	*13.*
57594	73087	107381	111111
46807	59763	95782	76543

14.	*15.*	*16.*
7890659	5346789	1234567
5769084	2537981	1159409

17.	*18.*	*19.*
80000000	76543021	58067359
78987685	67890807	37096789

20.	*21.*	*22.*
34567083	77665544	20000000
33075964	66778899	12345678

23.	*24.*	*25.*	*26.*
$5460.20	$4350.60	$5006.60	$1234.75
$4856.95	$3241.80	$4809.40	$143.85

27. I bought a cow for $30 and a horse for $60. I paid $75 down; how much have I yet to pay?

28. I bought a farm for $4520, and sold it for $4672. Find my gain.

29. My deposits in the bank have been \$50, \$175, \$94.75, \$281.50. I have checked out \$5, \$75.40, \$95.60, \$125. How much of the deposits have I yet in the bank?

30. I give a draft for \$2580.75 in payment of a debt of \$2499.90. How much change ought I to get?

31. How many years since the Declaration of Independence, 1776 to 1900?

32. In a school of 3254 pupils, 1576 are boys. How many girls are there?

33. I sold a man a bill of goods for \$247.50, and he paid me cash \$178.75, and a note for the balance. Find the amount for which the note was given.

34. I traded farms with my neighbor. His farm was valued at \$3240, and mine at \$2480. What difference should I pay him?

35. Mr. Brown bought a bill of goods amounting to \$1180.20, paying cash \$987.60. How much does he yet owe?

36. Mr. Johnson bought a farm for \$1786. He sold it at a loss of \$499.50. What did he get for it?

95. Subtraction of Compound Numbers.

EXAMPLES.

1. From 15 T. 6 cwt. 12 lb. 5 oz. take 6 T. 10 cwt. 8 lb. 14 oz.

Process:

T.	cwt.	lb.	oz.	
15	6	12	5	
6	10	8	14	
8	16	3	7	result.

Explanation: (1) 5 is smaller than 14. We must take 1 lb. from the 12 lb. and add it to the 5 oz. 1 lb. + 5 oz. = 21 oz. 21 − 14 = 7. Write the *7* below.

(2) There are only 11 lb. left in the minuend. (Why?) 11 − 8 = 3. Write the *3* below.

(3) 10 is larger than 6. (What shall we do?) 1 T. + 6 cwt. = 26 cwt. 26 − 10 = 16. Write the *16* below.

(4) There are 14 T. left in the minuend. (Why?) 14 − 6 = 8. Write the *8* below. *Result, 8 T. 16 cwt. 3 lb. 7 oz.*

NOTE.—Teacher should point out the similarity between this and *subtraction of simple numbers.*

2. From 1900 yr. 5 mo. 18 da. take 1896 yr. 6 mo. 10 da.

Process:

yr.	mo.	da.	
1900	5	18	
1896	6	10	
3	11	8	result.

NOTE.—Have the pupil explain.

EXERCISE LXXXVII.

Subtract:

1.

T.	cwt.	lb.	oz.
11	17	50	9
10	16	75	6

2.

yr.	mo.	da.	hr.	min.
50	9	20	16	20
35	11	16	21	10

3.

yd.	ft.	in.
12	2	8
9	1	10

4.

yr.	mo.	da.
1900	4	21
1877	9	7

5.

hr.	min.	sec.
17	45	30
12	50	37

6.

bu.	pk.	qt.	pt.
17	3	6	1
14	0	7	1

7.

gal.	qt.	pt.	gi.
36	2	1	2
17	0	1	3

8.

yr.	mo.	da.	hr.	min.	sec.
70	3	19	12	50	12
44	6	13	21	56	48

9. A note dated July 10, 1897, was paid September 23, 1899. How long was the time between these dates?

Process:

yr.	mo.	da.	
1899	9	23	
1897	7	10	
2	2	13	result.

Explanation: Write the latest date above. Write the number of the month: September, *9th* month; July, *7th* month. *The pupil should learn the number of each month in the year.*

10. Find the difference between the dates of the following sets: (1) Jan. 10, 1895, and July 17, 1894; (2) Aug. 5, 1894, and Feb. 1, 1898; (3) May 12, 1878, and April 10, 1892; (4) Oct. 7, 1888, and June 5, 1896.

11. A farmer takes to market 13 bu. 1 pk. of apples, but sells only 8 bu. 2 pk. How much has he left?

12. A pole is 5 yd. 2 ft. 6 in. long, but it was 7 yd. 1 ft. 8 in. long. How much has been cut off?

13. When I looked at my watch last it was 18 min. after 9 o'clock, but it is now 12 min. after 11 o'clock. How long has it been since I looked at my watch?

Form.

hr.	min.
11	12
9	18

NOTE.—18 minutes after 9 o'clock is 9 hr. 18 min. (from noon or midnight).

14. Out of a barrel of sugar containing 2 cwt. 36 lb., a merchant sells 1 cwt. 45 lb. How much sugar is left?

15. A milkman started out in the morning with 18 gal. of milk; he sold 12 gal. 2 qt. 1 pt. 3 gi. How much had he left?

16. From a barrel containing 35 gal. 2 qt. of molasses there are sold 27 gal. 3 qt. 1 pt. How much remained unsold?

17. A train leaves Philadelphia at 8 o'clock 20 min. p. m.; it reaches Baltimore at 11 o'clock 10 min. p. m. How long is the train in making the run?

18. A farmer weighed a load of hay on his wagon. The wagon and hay weighed 2 T. 2 cwt. 36 lb., but the wagon weighs 10 cwt. 44 lb. Find the weight of the hay.

III. MULTIPLICATION.

96. Review.

EXERCISE LXXXVIII.

1. What is *multiplication?* Define *multiplicand*, *multiplier*, *product.*

2. What is the sign of multiplication called? Should it be placed after or before the multiplier?

3. When the numbers are large, how do you place them for multiplication? Which figure of the multiplier do you use first? Which part of the multiplicand do you multiply first?

4. Do you "*carry*" in multiplication? Explain the process of multiplication, using this example: *37*×*342*.

5. Can you multiply 25 horses by $36? If so, explain the process. Can you multiply 25 horses by 36? Is the 36 abstract or concrete? Does the multiplier always have to be abstract? *Answer:* Yes, always.

Multiply:

6.	*7.*	*8.*	*9.*
34651	35980	87603	90643
324	246	304	519
10.	*11.*	*12.*	*13.*
84906	84881	97063	10876
643	987	567	957
14.	*15.*	*16.*	*17.*
70938	88709	50706	89768
507	709	395	476
18.	*19.*	*20.*	*21.*
75238	37659	70496	18796
946	886	593	947

22. 100×84659
23. 200×94306
24. 240×78461
25. 780×54369
26. 300×80600
27. 2400×7460
28. 700×90000
29. 3060×98765
30. 4080×70036
31. 9000×86479
32. 1200×89076
33. 1360×90080
34. 5400×87659
35. 1234×56789

36. *A* **Problem** *is a question proposed for solution.* (Commit.)

37. (1) What are 10 books worth?

(2) What are 10 books worth, at 50¢ each?

Is (1) a problem? Why?

Is (2) a problem? Why?

38. A mere question admits of an answer, but does it admit of a solution? What must there be besides a question to make a problem? (See Article 24.)

39. What are the parts of a problem? State in equations the parts of the problem in No. 37. Is the *basis* of a problem sometimes *understood* and not expressed? (See No. 44 below.)

40. What is an equation? What are its members called? (See Article 52.)

Give complete solution for each of the following:

41. A man bought a farm of 1260 acres, at $25 per acre. How much did he pay?

42. 2350 cattle sell at an average price of $34 per head. How much do they bring?

43. A book has 964 pages. How many pages in 74 such books?

44. Reduce 150 cwt. to pounds.

NOTE.—Basis is not stated. It is supposed that the pupil already knows that

$$1 \text{ cwt.} = 100 \text{ lb.}$$

45. A train averages 46 miles per hour for 23 hours. How far does it run in that time?

46. What will 246 books sell for, at $2.40 each?

47. A clock strikes 156 times in 1 day. How many times will it strike in 1 year of 365 days?

48. A wagon factory sells in a year 1264 wagons, at $52.50 each. What do the sales amount to for the year?

49. The shares of a certain corporation sell at $124.50 each. What will 25 shares cost me?

50. An Englishman in the United States gets a draft from home calling for 346 pounds of English money. Now, if 1

pound of English money is worth $4.85 of our money, how much United States money is his draft worth?

Basis: £1 = $4.85.

NOTE.—The *pound-mark*, £, is read *pound* or *pounds*, and is placed before the number after which it is to be read.

97. Multiplication of Compound Numbers.

EXAMPLES.

1. Multiply 3 bu. 1 pk. 5 qt. 1 pt. by 7.

Process.

bu.	pk.	qt.	pt.
3	1	5	1
			7
23	3	6	1 result.

Explanation: (1) 7 × 1 pt. = 7 pt. = 3 qt. 1 pt. Write the *1* below and carry the *3*.

(2) 7 × 5 qt. = 35 qt. 35 qt. + 3 qt. (carried) = 38 qt. = 4 pk. 6 qt. Write the *6* below, and carry the *4*.

(3) 7 × 1 pk. = 7 pk. 7 pk. + 4 pk. (carried) = 11 pk. = 2 bu. 3 pk. Write the *3* below, and carry the *2*.

(4) 7 × 3 bu. = 21 bu. 21 bu. + 2 bu. (carried) = 23 bu. Write the *23* below. *Result, 23 bu. 3 pk. 6 qt. 1 pt.*

NOTE.—The teacher should point out the similarity between this and *Multiplication of Simple Numbers.*

2. Multiply 27 yd. 2 ft. 9 in. by 9.

Process.

yd.	ft.	in.
27	2	9
		9
251	0	9 result.

Explanation: (1) 9 × 9 = 81. 81 in. = 6 ft. 9 in. Write the *9* below, and carry the *6*.

(2) 9 × 2 = 18. 18 ft. + 6 ft. = 24 ft. = 8 yd. Write the *0* below, and carry the *8*.

(3) 9 × 27 = 243. 243 yd. + 8 yd. = 251 yd. Write the *251* below. *Result, 251 yd. 9 in.*

3. Multiply 5 T. 8 cwt. 54 lb. 6 oz. by 8.

Process.

T.	cwt.	lb.	oz.
5	8	54	6
			8
43	8	35	0

NOTE.—Have the pupil explain this example.

EXERCISE LXXXIX.

Multiply:

	yr.	mo.	da.
1.	10	7	15
			5

	hr.	min.	sec.
2.	5	34	12
			6

	yd.	ft.	in.
3.	15	2	9
			8

4. Multiply 7 T. 11 cwt. 21 lb. 6 oz. by 11.

5. Multiply 15 yr. 11 mo. 25 da. by 7.

6. Multiply 36 hr. 40 min. 30 sec. by 4.

7. Multiply 12 bu. 1 pk. 3 qt. 1 pt. by 12.

8. Multiply 16 gal. 3 qt. 1 pt. 3 gi. by 5.

9. Multiply 12 gal. 1 qt. 2 gi. by 8.

10. Multiply 18 T. 13 cwt. 27 lb. by 9.

11. In a year a merchant sold 20 barrels of sugar of 2 cwt. 60 lbs. each. How much sugar did he sell?

12. A note was given for 1 yr. 4 mo. 20 da. By common consent the time of the note was extended 4 times that long. How long will the note run?

13. A stick 2 yd. 1 ft. 3 in. long is used in measuring a pole. The pole is 8 times as long as the stick. Find the length of the pole.

14. A grocer buys 9 barrels of syrup, each containing 44 gal. 2 qt. 1 pt. 3 gi. How much syrup does he buy?

15. A certain load of wheat contained 36 bu. 3 pk. What would 12 such loads contain?

IV. DIVISION.

98. Review.

EXERCISE XC.

1. Define *division*, *divisor*, *dividend*, *quotient*.

2. Explain the use of ÷, :. How do you read them?

3. How do you place the dividend and divisor for dividing?

When the divisor is less than 10, where do you place the quotient? When the divisor is large, where do you place the quotient?

4. How do you know when the quotient figure is too large? When too small?

5. Explain fully the process of dividing 8465 by 81. How do you dispose of the remainder?

6. Since division is a process of finding how many times one number contains another, can you divide \$250 by 25 hats? Must the dividend and divisor be of the same denomination?

7. If I want to find $\frac{1}{8}$ of \$372, what do I divide by? *Answer:* \$6. (See Third Application, p. 104.)

8. *A* **Ratio** *is the size of one number measured by another of the same kind.* (Commit.) The *Ratio* always tells how many times the *measured number* contains the *measure*.

9. Since a ratio tells how many times one number (the measured number) contains another number (the measure), how do we find a ratio? *Answer:* By division. (See Second Application, p. 104.)

10. What process would you use in simplifying each of the following expressions?

$$640 \div 8;\ 640 : 8;\ \tfrac{1}{8} \text{ of } 640;\ \tfrac{640}{8}.$$

Divide:

11.	*12.*	*13.*	*14.*
7)8505	8)9272	9)5787	6)2496

15. $52808 \div 644$
16. $52056 \div 723$
17. $52542 \div 834$
18. $46216 \div 872$
19. $51875 \div 625$

20. $24192 : 756$
21. $32004 : 762$
22. $45292 : 871$
23. $41676 : 453$
24. $54250 : 875$

25. $\frac{21045}{345}$
26. $\frac{30885}{435}$
27. $\frac{26406}{826}$
28. $\frac{33852}{872}$
29. $\frac{18832}{856}$

30. $\frac{1}{12}$ of 1728
31. $\frac{1}{21}$ of 4557
32. $\frac{1}{81}$ of 11935
33. $\frac{1}{41}$ of 25174
34. $\frac{1}{51}$ of 12087

35. 3468÷100
36. 48200÷200
37. 809100÷300
38. 6720÷120
39. 84360÷120
40. 22500÷150
41. 44100÷210
42. 894653÷963

Give complete solutions of the following (See Article 68 or Article 79):

43. I bought 43 calves for $537.50. Find the average cost of a calf.

44. Mr. Floyd bought 34 cattle for $833. How much was that per head?

45. In a field of corn there are 7800 hills in 65 rows. How many hills in a row?

46. 80 acres of land sell for $5760. How much is that per acre?

47. 86 men harvest 4300 acres of grain. How much is that for each man?

99. Division of Compound Numbers.

EXAMPLES.

1. Divide 27 bu. 3 pk. 3 qt. by 6.

Process.

	bu.	pk.	qt.	pt.
6)27	3	3	0	
4	2	4	1	result.

Explanation: (1) 27÷6=4, remainder 3. Write the *4* below, and reduce the 3 bu. to pecks.

(2) 3 bu.+3 pk.=15 pk. 15÷6=2, remainder 3. Write the *2* below, and reduce the 3 pk. to quarts.

(3) 3 pk.+3 qt.=27 qt. 27÷6=4, remainder 3. Write the *4* below, and reduce the 3 qt. to pints. 3 qt.=6 pt. 6÷6=1. Write the *1* below. *Result, 4 bu. 2 pk. 4 qt. 1 pt.*

NOTE.—Teacher should point out the similarity between this and *Division of Simple Numbers.*

2. Divide 112 T. 16 cwt. 59 lb. by 7.

Process.

	T.	cwt.	lb.
7)112	16	59	
16	2	37	result.

NOTE.—The pupil should explain this example.

EXERCISE XCI.

Divide:

1. yr. mo. da.
8)42 2 20

2. gal. qt. pt. gi.
5)64 3 0 3

3. T. cwt. lb. oz.
4)25 2 42 4

4. yd. ft. in.
12)94 0 0

5. hr. min. sec.
20)83 46 0

6. bu. pk. qt. pt.
9)41 1 7 1

7. bu. pk. qt.
7)33 2 6

8. da. hr. min. sec.
5)17 12 56 15

9. 490 bu. 2 pk. 4 qt. of grain is to be put in 100 sacks. How much must each sack hold?

10. A factory man puts 127 gal. 3 qt. 1 pt. 3 gi. of syrup into 63 buckets. How much does each bucket hold?

11. A man hauls 45 T. 18 cwt. of hay at 17 loads. What is the size of each load?

12. I bought 17 pieces of lumber of equal length, which measured altogether 161 yd. 1 ft. 6 in. long. How long was each piece?

V. REDUCTION OF DENOMINATE NUMBERS.

100. Review.

EXERCISE XCII.

1. Repeat the table of *Dry Measures*. What are these measures used for?

2. Repeat the table of *Liquid Measures*. What do we measure with these measures?

3. Repeat the table of *Avoirdupois Weights*. What are these weights used for?

4. Repeat the table of *Time Measures.*

5. Repeat the table of *Linear Measures.*

6. Reduce 5600 qt. to bu.

7. Reduce 6 bu. 3 pk. 4 qt. to pt.

8. Reduce 5 gal. 2 qt. 1 pt. 3 gi. to gi.

9. Reduce 5467 gi. to higher units.

10. Reduce 12 T. 15 cwt. 75 lb. to lb.

11. Reduce 57160 lb. to higher units.

12. Reduce 51840000 sec. to higher units.

13. Reduce 1 yr. 6 da. 2 hr. 2 min. 2 sec. to sec.

14. Reduce 12 yd. 2 ft. 8 in. to inches.

15. Reduce 5280 ft. to higher units.

16. What cost 5 bu. 3 pk. of peaches, at 30¢ per peck?

17. At 1¢ per pint, how many bushels of salt can be bought for $4.48?

18. At 5¢ per quart, how many gallons of milk can be bought for $3.40?

19. A man bales 10 cwt. 50 lb. of hay into bales of 75 lb. each. How many bales will he have?

NOTE.—Reduce the 10 cwt. 50 lb. to lb. before attempting the solution.

20. A merchant sells a barrel of molasses containing 54 gal. 3 qt. 1 pt., at 8¢ per pint. If it cost him 5¢ a pint, what does he gain on the barrel?

21. What will a 10-gallon can of oysters sell for, at 30¢ per quart?

22. A grocer buys 10 bu. of potatoes, at 40¢ per bu., and retails them at 15¢ per pk. How much does he gain on the lot?

23. I buy 1 cwt. 75 lb. of sugar, at 5¢ per lb., and sell it at 8¢ per lb.; I also spend $8.70 for milk, at 12¢ per gal., and sell it at 5¢ per qt. How much did I make on each, and which is the better deal?

24. The sun passes over 15 miles of the equator of the earth in 1 min. Over how many miles will it pass in 5 hr. 40 min.?

101. Other Tables of Weights.—You have already learned what weights are used at the *grocery store;* but if you go to the *drug store* for medicine, you will find a different set of weights, called **Apothecaries' Weights.** *Commit the table to memory.*

TABLE.

1 pound (lb.)=12 ounces (℥).
1 ℥=8 drams (ʒ).
1 ʒ=3 scruples (℈).
1 ℈=20 grains (gr.).

There is another table of weights which has the *grain, ounce and pound* of the same size as those of the *Apothecaries' Weights.* It is the table of **Troy Weights.** These weights are used in *jewelry stores,* and at mints, when the Government weighs gold and silver for making money. *Commit the table to memory.*

TABLE.

1 pound (lb.)=12 ounces (oz.).
1 oz.=20 pennyweights (pwt.).
1 pwt.=24 grains (gr.).

NOTE.—The Apothecaries' or Troy pound is smaller than the Avoirdupois pound, but the ounce is larger.

EXERCISE XCIII.

1. Reduce 17 lb. to ℈.
2. Reduce 17 lb. to pwt.
3. Reduce 1 lb. 1 ℥ 1 ʒ 1 ℈ 1 gr. to gr.
4. Reduce 1 lb. 1 oz. 1 pwt. 1 gr. to gr.
5. The silver dollar weighs 17 pwt. 4½ gr. How many grains does it weigh?
6. How many grains of quinine in 2ʒ?
7. The metal in a gold ring, weighing 4 pwt. 4 gr., is worth 5¢ per gr. What is the ring worth?

8. A druggist buys 10 lb. of indigo for $5, and sells it at 10¢ per oz. Find his gain on the 10 lb.

9. A druggist makes 3 scruples of opium into grain doses, which he sells at 5¢ per dose. What does he get for the opium?

10. **7000 gr. Troy = 1 lb. Avoirdupois.** Commit to memory.

11. Reduce 2 lb. avoirdupois to grains and higher units of the troy measures.

12. Reduce 58 lb. 4 ℥ to lb. avoirdupois.

102. Surveyors' Linear Measures. —These measures are used by surveyors in measuring distances, laying out land, roads, etc. *Commit the table to memory.*

TABLE.

1 mile (mi.) = 80 chains (ch.).

1 chain = { **4 rods (rd.),** or
66 ft., or
100 links (l.).

NOTE.—The *foot* in this table is the same as that in the *Common Linear Table*, p. 96. Thus, the two tables are connected, and form a complete table from *miles* to *inches*.

EXERCISE XCIV.

1. Reduce 6 mi. to rods.
2. Reduce 6 mi. to feet.
3. Reduce 6 mi. to links.
4. Reduce 6 mi. to inches.
5. Reduce 76500 l. to higher units.
6. Reduce 3840 rd. to mi.
7. Reduce 14080 yd. to mi.
8. Reduce 1848 ft. to rods.
9. Reduce 475200 in. to links.
10. Reduce 30 rods to inches.

103. Other Tables.—Writing pens, school crayons, eggs, and many other things, are numbered according to the following table. Commit it to memory.

TABLE.

1 great gross=12 gross.
1 gross=12 dozen (doz.).
1 doz.=12 things.

NOTE.—The term *score* is sometimes used.

1 score=20 things.

Sheets of paper are usually numbered by the following table. Commit it to memory.

TABLE.

1 bundle=2 reams.
1 ream=20 quires.
1 quire=24 sheets.

Grain and vegetables are often bought and sold by *weight-bushels* instead of the measured bushels. In the majority of States the bushel is of the following weight. Commit the table.

TABLE.

1 bu. of wheat=60 lb.
1 bu. of corn=56 lb.
1 bu. of oats=32 lb.
1 bu. of Irish potatoes=60 lb.
1 bu. of sweet potatoes=55 lb.

NOTE.—The *pounds* of the above table are *avoirdupois pounds*. These equivalents are not the same in all States, and the teacher should teach the pupil the table corrected for his own State.

EXERCISE XCV.

1. Reduce 2880 sheets of paper to bundles.

2. Reduce 7 bundles 7 quires 7 sheets to sheets.

3. Reduce 21 great gross to dozen.

4. I sell 5400 eggs, at 12½¢ per dozen. How much do I get for them?

5. A stationer buys a great gross of pens, at 50¢ a gross, and sells them 6 for 5¢. How much does he gain in the transaction?

6. Mr. Pile is 4 score 9 years old. Find his age in years.

7. I sold 1320 lb. of Irish potatoes, at 50¢ per bushel, and received in payment muslin, at 8¢ per yard. How many yards did I get for the potatoes?

Plan: (1) 1320 lb. = ( ) bu.?
(2) Cost of ( ) bu. = $( )?
(3) $( ) = cost of ( ) yd.?

8. Solve No. 7, if the potatoes were sweet potatoes.

9. I take to mill 1120 lb. of corn and exchange for flour. Corn is worth 25¢ per bu. and flour is worth $2.50 per hundredweight. How much flour do I get?

10. What will 5400 lb. of wheat sell for, at 62½¢ per bushel?

11. A certain book is made of paper whose sheets when folded make 8 leaves, or 16 pages. If the book contains 224 pages, how many such books can be made from 6 quires 10 sheets of the paper?

12. I exchange 384 lb. of oats, at 37½¢ per bu., for corn, at 50¢ per bu. How many bu. of corn do I get?

VI. DERIVED OPERATIONS.

104. Review Questions.

EXERCISE XCVI.

1. What is a *unit?*

NOTE.—Definitions should be thoroughly learned, and should be illustrated and discussed, if necessary. Do not permit a pupil to pass a definition without understanding it.

2. What is a *concrete unit?* an *abstract unit?*
3. What is an *integral unit?* a *fractional unit?*
4. What is a *number?*
5. What is an *abstract number?* a *concrete number?*
6. What is an *integer?* a *fraction?* a *mixed number?*
7. What is a *prime number?* a *composite number?*
8. What is a *factor?* a *prime factor?* a *common factor?*
9. When are numbers prime to each other?
10. When are the *divisor* and *quotient* factors?
11. What is the G. C. D. of two or more numbers?
12. What is the L. C. M. of two or more numbers?

105. Factoring.

EXERCISE XCVII.

1. Of what numbers is 2 a factor?

NOTE.—Numbers of which 2 is a factor are called *even numbers;* all others are called *odd numbers.*

2. Of what numbers is 3 a factor?
3. Of what numbers is 5 a factor?

NOTE.—If, by inspection, you find that 2, 3 or 5 is not a factor of a number, then, by division, try 7, 11, 13, or 17.

Find the prime factors of—

4. 231	*12.* 504	*20.* 1260	*28.* 5010
5. 400	*13.* 945	*21.* 2547	*29.* 3840
6. 351	*14.* 774	*22.* 1764	*30.* 3750
7. 550	*15.* 405	*23.* 2030	*31.* 8280
8. 486	*16.* 810	*24.* 4500	*32.* 3333
9. 612	*17.* 234	*25.* 3264	*33.* 5740
10. 850	*18.* 855	*26.* 5850	*34.* 4851
11. 702	*19.* 900	*27.* 6720	*35.* 5625

106. Finding the Greatest Common Divisor by Factoring.— See Article 75.

EXERCISE XCVIII.

1. What factors does the G.C.D. of two or more numbers contain?

Find the G. C. D. of—

2. 420, 945, 315.	*8.* 84, 108, 144.
3. 75, 125, 175.	*9.* 288, 132, 96.
4. 162, 81, 243.	*10.* 210, 315, 105.
5. 105, 85, 30.	*11.* 34, 85, 510.
6. 90, 36, 135.	*12.* 288, 36, 72.
7. 329, 56, 77.	*13.* 121, 847, 363.

107. Finding the Greatest Common Divisor by Division.

PRINCIPLES.

I. *Every factor of a number is also a factor of every multiple of that number.* Thus:

Since 2 and 3 are factors of 6, they are also factors of 12, 18, 24, 30, 36, etc.

NOTE.—A multiple of a number contains that number an integral number of times.

II. *Every common factor of two numbers is also a factor of their sum and of their difference.* Thus:

Since 4 is a common factor of 12 and 32, it is also a factor of 32+12, or 44; it is also a factor of 32−12, or 20.

III. *The Greatest Common Divisor of two numbers is the Greatest Common Divisor of either of them and their sum.* Thus:

Since 6 is the G. C. D. of 18 and 42, it is also the G. C. D. of 18 and 18+42, or 60; 6 is also the G. C. D. of 42 and 60.

Commit the above principles to memory.

EXAMPLES.

1. Find the G. C. D. of 254 and 381.

Process.

$$\begin{array}{r} 1 \\ 254\overline{)381} \\ 254 \quad 2 \\ \hline 127)\ 254 \\ 254 \\ \hline \end{array}$$

Explanation: Since 127 is the G. C. D. of 127 and 254, it is also the G. C. D. of 254 and 381 (Prin. III). ∴ the required G. C. D. is 127.

NOTE.—This plan or process of finding the G. C. D. is to divide the larger number by the smaller; then divide the divisor by the remainder; then continue to divide the last divisor by the last remainder, until there is no remainder. The last divisor is the required G. C. D.

2. Find the G. C. D. of 221, 364, and 5512.

Process.

(1)

$$\begin{array}{r} 15 \\ 364\overline{)5512} \\ 364 \\ \hline 1872 \\ 1820 \quad 7 \\ \hline 52)364 \\ 364 \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 4 \\ 52\overline{)221} \\ 208 \quad 4 \\ \hline 13)52 \\ 52 \\ \hline \end{array}$$

∴ the required G.C.D. is 13.

Explanation: (1) The G. C. D. of 364 and 5512 is 52. But 52 is not a factor of 221. (2) The G. C. D. of 52 and 221 is 13. Now, 52 contains all factors common to 364 and 5512; 13 is the only factor common to 52 and 221. Therefore 13 is the G. C. D. of 221, 364, and 5512.

NOTE.—Division is employed in finding the G. C. D. when the numbers are large.

EXERCISE XCIX.

Find the G. C. D. of—

1. 686, 1869.
2. 1485, 915.
3. 1313, 4121.
4. 1649, 5406.
5. 805, 3289.
6. 697, 789.
7. 1668, 3058.
8. 3070, 2149.
9. 906, 2265.
10. 710, 1065, 1207.
11. 869, 1111, 671.
12. 380, 779, 1577.

108. Finding the Least Common Multiple.—

Review Article 76.

PRINCIPLE.

The Least Common Multiple of two or more numbers is a multiple of all the factors of those numbers.

EXAMPLE.

Find the L. C. M. of 21, 15, 45.

Process.

$3)21,\ 15,\ 45$

$7 \quad\quad 15$

$3 \times 7 \times 15 = 315$, L. C. M.

Explanation: The 15 may be canceled from the process, for it is a factor of 45.

EXERCISE C.

Find the L. C. M. of —

1. 26, 39, 78.
2. 35, 56, 63.
3. 48, 80, 120.
4. 75, 125, 240.
5. 160, 24, 230.
6. 150, 60, 120.
7. 135, 60, 90.
8. 140, 77, 35.
9. 12, 18, 24, 36.
10. 6, 18, 48, 80.
11. 9, 15, 27, 120.
12. 75, 30, 16, 10.
13. 128, 72, 240, 100.
14. 132, 77, 60, 45.
15. 18, 75, 48, 120.
16. 42, 56, 63, 135.

VII. PROPORTION.

109. Review.

EXERCISE CI.

1. Illustrate what you mean by *indicated ratio;* by *numerical ratio.*

2. Define *antecedent* and *consequent* as used in ratio.

3. What is *proportion?* Give an example.

4. How many terms in a proportion? Which terms are called *means?* which *extremes?*

5. If one of the *means* of a proportion is missing, how can it be found?

6. If one of the *extremes* of a proportion is missing, how can it be found?

7. What are the parts of a problem?

8. Is the *form* of the equation the same for both parts of a problem?

9. Does the *Question* depend upon the condition stated in the *Basis?*

10. What is the *relation* existing between the members of two equations of the same form and depending upon the same condition?

Find the missing term:

11. $\frac{1}{2} : 10 :: 7 : (\ )$?

12. $(\ ) : 50 :: 210 : 700$?

13. $180 : 150 :: (\ ) : 225$?

14. $\frac{4}{5} : \frac{7}{8} :: \frac{8}{15} : (\ )$?

15. $3\frac{1}{2} : (\ ) :: 14 : 18$?

16. $(\ ) : 90 :: \frac{5}{7} : 5\frac{5}{14}$?

17. $15 : (\ ) :: \frac{3}{10} : 1$?

18. $12\frac{1}{2} : 87\frac{1}{2} :: (\ ) : 28$?

Solve the following problems:

19. What will 37 typewriters sell for, at \$90 each?

20. I paid \$168 for 24 law books. Find the price per book.

21. A man sells 11 chairs for $33. At that price, what will 2 dozen chairs bring?

22. I can buy 8 desks for $54. How many can I buy for $486?

23. Reduce 75 T. to hundredweight.

24. I invested $500 and gained $350. At that rate, what would I have gained on $1800?

25. A bill of exchange on London for £100 cost me $485. What would a bill for £500 have cost me?

26. If a bill of exchange for £320 sells for $1552, how large a bill can be bought for $3298?

27. I had $1400 invested in a speculation, and lost $200. At that rate, what would I have lost on $1792?

28. I had a certain amount of money invested, and gained $480. If I had invested $500, I would have gained but $200. How much did I invest?

29. A lawyer charges $40 for collecting a debt of $400. At that rate, what will he charge for collecting a debt of $1260?

30. A lawyer charges $64 for collecting a debt of $800. At the same rate, he charges $96 for collecting another debt. Find the amount of the second debt.

31. If the tax on $500 is $10, at that rate, what is the tax on $1860?

32. If I pay $84 tax on a piece of land assessed at $4200, at that rate, what is the value of land on which I pay $98 tax?

33. If $600 bear $24 interest in a certain time, at that rate, how much interest will $950 bear in the same time?

34. If 8 engines draw 60 cars, how many engines will be needed to draw 100 cars?

Question: Force reqd. to draw 100 cars = ( ) engines?

35. If the interest on $450 for a certain time is $27, how much money will produce $42 in the same time and at the same rate?

Question: $42 = interest on $( )?

36. If 10 men stack a wheat crop of 200 acres in a certain time, how many men can stack 160 acres in the same time?

37. Five men can plow 90 acres of land in a week. How many acres can 12 men plow in a week?

38. A certain company has $10000 capital, which gains in one year $4000. What is Mr. Johnson's share of the gain, if he owns $2500 of the capital?

Parts: (1) Gain on $2500 = $( )? (Question.)
(2) Gain on $10000 = $4000. (Basis.)

39. In the company of No. 38, Mr. Brown received as his gain $1200. What part of the capital does he own?

40. If 12 acres of land will produce 480 bu. of corn, at that rate, how many bu. will 75 acres produce?

VIII. COMMON FRACTIONS.

110. Review.

EXERCISE CII.

1. What is a *fraction?* A *common fraction?*

2. How many *terms* in a fraction? Name and define each.

3. What is an *improper fraction?*

4. What is the effect on the value of a fraction, if we multiply both numerator and denominator by the same number? (See Article 81.) Illustrate your answer by multiplying both terms of $\frac{3}{5}$ by 2.

5. What is the effect on the value of a fraction, if we divide both numerator and denominator by the same number? (See Article 82.) Illustrate your answer by dividing both terms of $1\frac{8}{2}$ by 4.

6. Is a common fraction an expression of division? Which term of the fraction is the *dividend?* Which the *divisor?*

111. Reducing Fractions to Higher Terms.—

Review Article 81.

EXERCISE CIII.

Oral drill:

NOTE.—Have the pupil read the exercise from the book, and then say (in Ex. 1), "3 in 9 *three* times, therefore we must multiply both terms of the fraction by 3. 3×2 are 6; 3×3 are 9; result, $\frac{6}{9}$."

1. Reduce $\frac{2}{3}$ to 9ths.
2. Reduce $\frac{1}{5}$ to 20ths.
3. Reduce $\frac{5}{6}$ to 18ths.
4. Reduce $\frac{3}{4}$ to 16ths.
5. Reduce $\frac{3}{5}$ to 25ths.
6. Reduce $\frac{4}{7}$ to 35ths.
7. Reduce $\frac{5}{7}$ to 56ths.
8. Reduce $\frac{3}{8}$ to 64ths.
9. Reduce $\frac{5}{3}$ to 81sts.
10. Reduce $\frac{5}{12}$ to 60ths.

Written drill:

11. Reduce $\frac{5}{9}$ to 108ths.
12. Reduce $\frac{6}{22}$ to 264ths.
13. Reduce $1\frac{1}{7}$ to 153rds.
14. Reduce $1\frac{7}{13}$ to 325ths.
15. Reduce $1\frac{2}{25}$ to 425ths.
16. Reduce $1\frac{7}{36}$ to 252nds.
17. Reduce $1\frac{5}{9}$ to 209ths.
18. Reduce $2\frac{5}{7}$ to 324ths.
19. Reduce 5 to 57ths.
20. Reduce $6\frac{1}{3}$ to 15ths.
21. Reduce $15\frac{3}{7}$ to 42nds.
22. Reduce $24\frac{5}{19}$ to 57ths.

112. Reducing Fractions to Lowest Terms.—

Review Article 82.

EXERCISE CIV.

Oral drill: Reduce to lowest terms—

NOTE.—Have the pupil read the exercise, and then say (in Ex. 1), "We can divide by 5. 5 in 5 once; 5 in 10 twice; result, $\frac{1}{2}$."

1. $\frac{5}{10}$
2. $\frac{6}{18}$
3. $\frac{6}{9}$
4. $\frac{12}{30}$
5. $\frac{8}{20}$
6. $\frac{9}{27}$
7. $\frac{18}{54}$
8. $\frac{15}{35}$
9. $\frac{25}{40}$
10. $\frac{20}{50}$
11. $\frac{33}{44}$
12. $\frac{27}{36}$
13. $\frac{14}{42}$
14. $\frac{48}{80}$
15. $\frac{56}{84}$

Written drill: Reduce to lowest terms—

16. $\frac{27}{135}$	*21.* $\frac{108}{360}$	*26.* $\frac{324}{360}$
17. $\frac{84}{204}$	*22.* $\frac{121}{209}$	*27.* $\frac{441}{840}$
18. $\frac{81}{342}$	*23.* $\frac{231}{385}$	*28.* $\frac{184}{384}$
19. $\frac{48}{405}$	*24.* $\frac{130}{273}$	*29.* $\frac{165}{1850}$
20. $\frac{70}{350}$	*25.* $\frac{240}{500}$	*30.* $\frac{350}{735}$

113. Reducing Mixed Numbers to Improper Fractions.—Review Article 83.

EXERCISE CV.

Oral drill: Reduce to improper fractions—

NOTE.—Have the pupil read the exercise, and then say (in Ex. 1), "3×5 are 15, and 2 are 17; result, $\frac{17}{3}$."

1. $5\frac{2}{3}$	*6.* $12\frac{1}{2}$	*11.* $11\frac{1}{8}$
2. $4\frac{3}{7}$	*7.* $10\frac{3}{5}$	*12.* $11\frac{3}{5}$
3. $3\frac{5}{8}$	*8.* $9\frac{4}{7}$	*13.* $12\frac{2}{3}$
4. $7\frac{2}{5}$	*9.* $8\frac{8}{9}$	*14.* $12\frac{3}{4}$
5. $9\frac{3}{8}$	*10.* $9\frac{5}{9}$	*15.* $9\frac{5}{8}$

Written drill: Reduce to improper fractions—

16. $26\frac{5}{17}$	*21.* $89\frac{7}{24}$	*26.* $120\frac{5}{44}$
17. $34\frac{4}{19}$	*22.* $95\frac{9}{37}$	*27.* $265\frac{5}{29}$
18. $45\frac{5}{17}$	*23.* $87\frac{1}{29}$	*28.* $378\frac{1}{56}$
19. $73\frac{3}{31}$	*24.* $79\frac{3}{41}$	*29.* $607\frac{4}{25}$
20. $76\frac{1}{19}$	*25.* $93\frac{8}{59}$	*30.* $598\frac{31}{400}$

114. Reducing Improper Fractions to Integers or Mixed Numbers.—Review Article 84.

EXAMPLE.

Reduce $\frac{488}{28}$ to mixed number.

Process.

$$\begin{array}{r} 17\frac{12}{28}, \text{ result.} \\ 28\overline{)488} \\ \underline{28} \\ 208 \\ \underline{196} \\ \frac{12}{28}=\frac{3}{7}. \end{array}$$

NOTE.—The fraction $\frac{12}{28}$ should be reduced to its lowest terms, $\frac{3}{7}$.

EXERCISE CVI.

Oral drill: Reduce to integers or mixed numbers—

NOTE.—Have the pupil read the exercise, and say (in Ex. 1), "3 in 21 seven times; result, 7."

1. $\frac{21}{3}$	6. $\frac{75}{7}$	11. $\frac{98}{5}$
2. $\frac{23}{4}$	7. $\frac{84}{9}$	12. $\frac{125}{10}$
3. $\frac{36}{7}$	8. $\frac{76}{8}$	13. $\frac{134}{9}$
4. $\frac{48}{6}$	9. $\frac{96}{9}$	14. $\frac{245}{8}$
5. $\frac{63}{8}$	10. $\frac{77}{8}$	15. $\frac{264}{7}$

Written drill: Reduce to integers or mixed numbers—

16. $\frac{175}{12}$	21. $\frac{763}{24}$	26. $\frac{1260}{48}$
17. $\frac{384}{15}$	22. $\frac{849}{37}$	27. $\frac{837}{19}$
18. $\frac{596}{27}$	23. $\frac{976}{126}$	28. $\frac{947}{39}$
19. $\frac{742}{36}$	24. $\frac{794}{84}$	29. $\frac{1596}{108}$
20. $\frac{856}{29}$	25. $\frac{740}{21}$	30. $\frac{34836}{245}$

115. Reducing Fractions to Least Common Denominator.—Review Article 85.

EXERCISE CVII.

1. What is the L. C. D. of two or more fractions?
2. When do we need to reduce fractions to L. C. D.?

Answer: When addition or subtraction is to be performed.

Oral drill: Reduce to L. C. D.—

NOTE.—Have the pupil read the exercise, and say (in Ex. 1), "The L. C. M. of 2 and 3 is 6." Then reduce $\frac{1}{2}$ and $\frac{1}{3}$ to 6ths, as in Article 111.

1. $\frac{1}{2}$, $\frac{1}{3}$.	6. $\frac{3}{4}$, $\frac{2}{5}$.	11. $\frac{3}{7}$, $\frac{1}{2}$.
2. $\frac{2}{3}$, $\frac{3}{4}$.	7. $\frac{2}{3}$, $\frac{5}{7}$.	12. $\frac{2}{3}$, $\frac{1}{8}$.
3. $\frac{1}{2}$, $\frac{3}{4}$.	8. $\frac{1}{4}$, $\frac{5}{8}$.	13. $\frac{3}{4}$, $\frac{5}{7}$.
4. $\frac{1}{4}$, $\frac{5}{6}$.	9. $\frac{5}{9}$, $\frac{5}{6}$.	14. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
5. $\frac{2}{3}$, $\frac{5}{6}$.	10. $\frac{3}{5}$, $\frac{7}{10}$.	15. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

Written drill: Reduce to L. C. D.—

16. $\frac{3}{5}$, $\frac{5}{16}$, $\frac{9}{35}$.
17. $\frac{2}{7}$, $\frac{9}{14}$, $\frac{1}{27}$.
18. $\frac{8}{15}$, $\frac{12}{17}$, $\frac{14}{255}$.
19. $\frac{23}{48}$, $\frac{5}{9}$, $\frac{14}{15}$.
20. $\frac{5}{34}$, $\frac{9}{17}$, $\frac{18}{85}$.
21. $\frac{11}{21}$, $\frac{15}{49}$, $\frac{6}{35}$.
22. $\frac{17}{24}$, $\frac{5}{36}$, $\frac{19}{60}$.
23. $\frac{21}{25}$, $\frac{13}{50}$, $\frac{11}{75}$.
24. $\frac{1}{3}$, $\frac{2}{7}$, $\frac{5}{9}$, $\frac{11}{21}$.
25. $\frac{3}{5}$, $\frac{5}{16}$, $\frac{6}{25}$, $\frac{8}{8}$.
26. $\frac{5}{8}$, $\frac{11}{16}$, $\frac{5}{24}$, $\frac{7}{12}$.
27. $\frac{5}{27}$, $\frac{2}{3}$, $\frac{13}{36}$, $\frac{7}{18}$.
28. $\frac{4}{9}$, $\frac{5}{18}$, $\frac{3}{5}$, $\frac{23}{45}$.
29. $\frac{6}{11}$, $\frac{12}{55}$, $\frac{9}{22}$, $\frac{15}{44}$.
30. $\frac{13}{30}$, $\frac{13}{15}$, $\frac{13}{45}$, $\frac{13}{60}$.
31. $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$.

116. Addition of Fractions.—Review Article 86.

EXERCISE CVIII.

Oral drill: Find the value of—

NOTE.—Have the pupil (1) read exercise, (2) reduce fractions to L. C. D., (3) add, and (4) simplify, if necessary.

1. $\frac{1}{2}+\frac{1}{3}$
2. $\frac{1}{2}+\frac{1}{4}$
3. $\frac{1}{2}+\frac{3}{4}$
4. $\frac{1}{3}+\frac{3}{4}$
5. $\frac{2}{3}+\frac{3}{4}$
6. $\frac{1}{2}+\frac{1}{6}$
7. $\frac{2}{3}+\frac{1}{6}$
8. $\frac{1}{3}+\frac{5}{6}$
9. $\frac{1}{2}+\frac{5}{7}$
10. $\frac{1}{3}+\frac{3}{7}$
11. $4\frac{2}{3}+\frac{5}{7}$
12. $\frac{3}{4}+6\frac{1}{8}$
13. $2\frac{1}{2}+3\frac{1}{5}$
14. $4\frac{3}{4}+7\frac{5}{6}$
15. $6\frac{2}{3}+5\frac{1}{9}$

Written drill: Find the value of—

16. $1\frac{1}{12}+\frac{5}{16}$
17. $1\frac{3}{15}+\frac{7}{9}$
18. $\frac{1}{27}+\frac{23}{36}$
19. $17\frac{1}{18}+19\frac{5}{8}$
20. $22\frac{2}{45}+33\frac{1}{15}$
21. $\frac{3}{7}+\frac{5}{21}+\frac{13}{24}$
22. $\frac{26}{45}+\frac{13}{30}+\frac{8}{15}$
23. $\frac{17}{30}+\frac{19}{24}+\frac{1}{60}$
24. $\frac{3}{13}+\frac{5}{11}+\frac{4}{7}$
25. $\frac{7}{50}+\frac{19}{75}+\frac{11}{125}$
26. $6\frac{5}{18}+9\frac{4}{27}+12\frac{5}{9}$
27. $26+8\frac{3}{40}+7\frac{7}{60}$
28. $1\frac{23}{50}+35+10\frac{11}{35}$
29. $72\frac{31}{48}+18\frac{29}{32}+79\frac{17}{24}$
30. $246\frac{23}{72}+186\frac{19}{36}+759\frac{13}{18}$

117. Subtraction of Fractions.—Review Article 87.

EXERCISE CIX.

Oral drill: Find the value of—

NOTE.—Have the pupil (1) read exercise, (2) reduce fractions to L.C. D., (3) subtract, and (4) simplify, if necessary.

1. $\frac{1}{2}-\frac{1}{3}$
2. $\frac{1}{2}-\frac{1}{4}$
3. $\frac{3}{4}-\frac{2}{3}$
4. $\frac{3}{4}-\frac{1}{5}$
5. $\frac{3}{4}-\frac{2}{5}$
6. $\frac{4}{5}-\frac{1}{4}$
7. $\frac{1}{4}-\frac{1}{6}$
8. $\frac{5}{6}-\frac{3}{4}$
9. $\frac{5}{6}-\frac{1}{2}$
10. $\frac{7}{8}-\frac{3}{4}$
11. $1\frac{7}{8}-1\frac{1}{2}$
12. $3\frac{1}{3}-2\frac{1}{5}$
13. $6\frac{5}{7}-3\frac{1}{3}$
14. $5\frac{5}{9}-4\frac{2}{3}$
15. $8\frac{3}{5}-2\frac{3}{4}$

Written drill: Find the value of—

16. $1\frac{1}{15}-\frac{3}{45}$
17. $1\frac{7}{32}-\frac{8}{24}$
18. $\frac{29}{40}-\frac{3}{11}$
19. $\frac{35}{29}-\frac{2}{3}$
20. $\frac{7}{39}-\frac{4}{65}$
21. $9\frac{3}{14}-\frac{7}{35}$
22. $7\frac{8}{9}-5\frac{17}{45}$
23. $12\frac{5}{19}-11\frac{17}{52}$
24. $25-18\frac{13}{29}$
25. $36-19\frac{49}{57}$
26. $37\frac{15}{39}-25\frac{19}{78}$
27. $75\frac{3}{5}-35\frac{17}{21}$
28. $86\frac{15}{19}-47\frac{31}{32}$
29. $29\frac{12}{13}-28\frac{18}{19}$
30. $425\frac{53}{60}-349\frac{89}{90}$

118. Multiplication of Fractions.—Review Article 88.

EXERCISE CX.

Oral drill: Find the value of—

NOTE.—Have the pupil read the exercise, and say (in Ex. 1), "Once 3 is 3; 2×5 are 10; result, $\frac{3}{10}$." The pupil should employ the principle of cancellation where possible.

1. $\frac{1}{2}$ of $\frac{3}{5}$
2. $\frac{1}{2}$ of $\frac{5}{6}$
3. $\frac{2}{3}$ of $\frac{4}{5}$
4. $\frac{1}{5}$ of $\frac{3}{7}$
5. $2\times\frac{4}{9}$
6. $\frac{5}{7}\times 11$
7. $\frac{8}{9}\times\frac{3}{5}$
8. $\frac{6}{7}\times 1\frac{1}{2}$
9. $\frac{8}{8}\times\frac{4}{15}$
10. $\frac{2}{3}$ of $1\frac{1}{2}$
11. $\frac{8}{11}\times\frac{5}{12}$
12. $\frac{1}{8}$ of $6\times\frac{1}{2}$
13. $\frac{2}{3}$ of $9\times\frac{1}{10}$
14. $7\times\frac{5}{14}$ of $\frac{3}{7}$
15. $\frac{2}{3}$ of $\frac{3}{4}$ of 8

Written drill: Find the value of—

16. $\frac{1}{7}\frac{1}{}$ of $\frac{5}{21}\frac{1}{}\times\frac{5}{18}$
17. $\frac{4}{21}\times\frac{9}{26}$ of $1\frac{3}{8}$
18. $\frac{1}{8}\frac{5}{2}\times\frac{2}{5}\frac{4}{0}\times\frac{3}{3}\frac{2}{5}$
19. $\frac{1}{3}\frac{6}{5}$ of $\frac{1}{4}\frac{4}{5}$ of $\frac{9}{3}\frac{9}{2}$
20. $\frac{1}{2}\frac{2}{5}\times\frac{1}{2}\frac{3}{4}\times\frac{1}{2}\frac{5}{6}$
21. $25\times\frac{1}{5}\frac{3}{0}\times5\frac{3}{26}$
22. $\frac{5}{1}\frac{}{2}$ of $17\frac{1}{2}\times\frac{1}{21}$
23. $\frac{1}{4}\frac{3}{2}\times\frac{1}{6}\frac{5}{4}\times7\frac{9}{17}$
24. $\frac{7}{1}\frac{}{2}$ of $64\times3\frac{3}{16}$
25. $7\frac{3}{10}\times\frac{39}{146}\times2\frac{4}{13}$
26. $46\frac{3}{4}\times7\frac{13}{17}\times9\frac{4}{9}$
27. $24\frac{5}{9}\times12\frac{6}{7}\times12\frac{2}{15}$

119. Division of Fractions.—Review Article 90.

EXERCISE CXI.

1. What does inverting a divisor show?

2. In division of fractions, how do you proceed after inverting the divisor?

Oral drill: Find the value of—

NOTE.—Have the pupil read the exercise, and say (in Ex. 1), "$\frac{3}{4}\div\frac{1}{5}=\frac{3}{4}\times\frac{5}{1}=\frac{15}{4}$, or $3\frac{3}{4}$, result."

1. $\frac{3}{4}\div\frac{1}{5}$
2. $\frac{1}{2}\div\frac{1}{4}$
3. $\frac{1}{3}\div\frac{1}{6}$
4. $\frac{3}{4}\div\frac{1}{8}$
5. $\frac{2}{3}\div\frac{1}{9}$
6. $\frac{3}{7}\div\frac{9}{14}$
7. $\frac{2}{5}\div\frac{3}{10}$
8. $\frac{5}{9}\div5$
9. $6\div\frac{7}{12}$
10. $8\frac{1}{2}\div\frac{1}{4}$
11. $\frac{3}{8}\div\frac{3}{4}$
12. $\frac{3}{7}\div\frac{4}{21}$
13. $\frac{3}{5}\div\frac{7}{15}$
14. $\frac{3}{8}\div\frac{7}{16}$
15. $7\frac{1}{2}\div2\frac{2}{5}$

Written drill: Find the value of—

16. $16\div\frac{5}{8}$
17. $14\div\frac{8}{21}$
18. $\frac{1}{36}\div22$
19. $\frac{1}{19}\frac{5}{}\div10$
20. $\frac{5}{64}\frac{1}{}\div\frac{1}{7}\frac{7}{2}$
21. $\frac{4}{7}\frac{5}{7}\div\frac{9}{88}$
22. $5\frac{5}{8}\div3\frac{9}{27}$
23. $\frac{15}{121}\div\frac{45}{88}$
24. $17\frac{2}{3}\div\frac{7}{106}$
25. $6\frac{1}{8}\div7\frac{7}{13}$
26. $12\frac{9}{16}\div8\frac{17}{23}$
27. $8\frac{11}{20}\div14\frac{7}{17}$
28. $\frac{16\frac{13}{24}}{14\frac{5}{16}}$
29. $\frac{24\frac{4}{9}}{12\frac{4}{27}}$
30. $\frac{31\frac{31}{87}}{41\frac{41}{47}}$

IX. GENERAL SUBJECTS.

120. Use of Signs.—When a numerical expression is separated into parts by the signs, plus (+) and minus (−), or either of them, these parts are **Terms.** As,

$$9+\tfrac{1}{2}-3+7.$$

NOTE.—This expression has four terms: 9, $\frac{1}{2}$, 3, and 7.

The signs of multiplication (× and "of") and division (÷) do not separate an expression into terms; they separate only the parts of a term. Thus,

$$5\times 6-\tfrac{1}{6} \text{ of } 12-21\div 7.$$

NOTE.—This expression has but three terms:
5×6, $\frac{1}{6}$ of 12, and $21\div 7$.

EXAMPLE.

Simplify $\frac{5}{16}\div\frac{7}{8}+12\times 10-\frac{5}{9}$ of 54.

Plan: (1) Simplify each term, and (2) perform the addition and subtraction. (See p. 32, mixed drill.)

Process: (1) $\frac{5}{16}\div\frac{7}{8}=\frac{5}{\cancel{16}_{2}}\times\frac{\cancel{8}}{7}=\frac{5}{14}$.

(2) $12\times 10=120$.

(3) $\frac{5}{9}$ of $54=30$.

(4) $\frac{5}{14}+120-30=90\frac{5}{14}$, result.

Or if the numbers are small enough for the work to be performed mentally, then this form is better:

Process: (1) $\frac{5}{16}\div\frac{7}{8}+12\times 10-\frac{5}{9}$ of $54=$
$\frac{5}{14}+120-30=90\frac{5}{14}$, result.

EXERCISE CXII.

Oral drill: Simplify—

1. $\frac{4}{5}-\frac{1}{4}$ of $\frac{3}{5}$.
2. $\frac{3}{4}\div\frac{5}{8}-\frac{4}{5}$.
3. $\frac{1}{2}$ of $\frac{2}{5}+\frac{3}{5}$ of $\frac{5}{6}$.
4. $\frac{2}{3}+\frac{2}{7}$ of $7-1\frac{1}{3}$.
5. $15\div 3+\frac{3}{8}\div\frac{5}{16}$.
6. $2\frac{1}{3}\times\frac{5}{7}-\frac{1}{2}$ of $\frac{2}{3}$.
7. $\frac{5}{9}\div 3\frac{1}{3}+\frac{2}{3}$ of $\frac{1}{2}$.
8. $7\frac{1}{2}-\frac{3}{5}$ of $2\frac{1}{2}$.
9. $18\div 6+\frac{3}{7}$ of $\frac{7}{9}-\frac{1}{4}\times\frac{2}{3}$.
10. $\frac{2}{3}$ of $12-\frac{1}{6}\div\frac{2}{3}+\frac{5}{9}\times\frac{3}{10}$.

Written drill: Simplify—

11. $7\times11-\frac{5}{16}$ of 64.

12. $17\frac{3}{4}+\frac{3}{11}$ of $13\frac{3}{4}$.

13. $220\div75-\frac{2}{7}\div\frac{5}{28}+6\frac{3}{10}$.

14. $128\div6\frac{2}{5}+\frac{3}{10}$ of $13\frac{1}{3}-4\frac{4}{5}\times\frac{5}{12}$.

15. $7\times9-\frac{6}{17}\times\frac{34}{63}+\frac{9}{25}$ of $6\frac{9}{11}$.

16. $7\times5\times4-\frac{25}{42}\div\frac{25}{72}-\frac{3}{14}$ of $\frac{21}{33}$ of $\frac{11}{18}$.

17. $\frac{35}{64}\div\frac{5}{192}-\frac{8}{25}$ of $\frac{45}{32}+\frac{9}{20}\times\frac{4}{9}\times\frac{5}{8}$.

18. $8\frac{1}{5}\div5\frac{1}{8}-\frac{7}{17}\times\frac{34}{49}+1\frac{1}{12}\times8\frac{4}{7}$.

19. $71\frac{1}{7}\times\frac{28}{249}-\frac{3}{5}$ of $8\frac{2}{11}+12\frac{4}{21}\div\frac{7}{32}$.

20. $29\frac{7}{12}\times\frac{24}{71}$ of $\frac{5}{18}+825\div5\frac{10}{18}-\frac{4}{9}$ of $\frac{33}{40}$ of $10\frac{10}{11}$.

121. Fractional Denominate Numbers.—Review Article 89, p. 137.

EXERCISE CXIII.

1. Reduce $\frac{3}{5}$ oz. to the fraction of a cwt.

2. Reduce $\frac{7}{18}$ min. to the fraction of a day.

3. Reduce $\frac{1}{28}$ lb. avoirdupois to the fraction of an oz.

4. Reduce $\frac{1}{320}$ bu. to the fraction of a pint.

5. Reduce $\frac{3}{4}$ bu. $\frac{1}{16}$ pk. $2\frac{1}{2}$ qt. to pt.

6. Reduce 2 pk. 4 qt. to the fraction of a bu.

7. Reduce $\frac{1}{28}$ lb. troy to pwt.

8. Reduce $\frac{4}{5}$ oz. troy to lower integral units.

9. Reduce 7 lb. $8\frac{1}{4}$ oz. avoirdupois to the fraction of a cwt.

10. Reduce $\$\frac{3}{350}$ to the fraction of a cent.

11. Reduce $\frac{2}{3}$ da. $\frac{3}{4}$ hr. to min.

12. Reduce 24 da. 8 hr. to the fraction of a yr.

122. Business Forms. A **Bill of Goods** is a written statement of articles sold, together with the price of each and the entire price.

[A BILL.]

SAN FRANCISCO, CAL., Oct. 3, 1900.

MR. H. G. ADAMS,
OAKLAND, CAL.,

To J. H. EDWARDS, DR.

TERMS, on %.

Sept.	3	1 sack of flour	$1	60
"	8	12 lb. beans, @ 5¢		60
"	12	12 lb. sugar, @ 8⅓¢	1	
"	12	5 lb. coffee, @ 25¢	1	25
"	17	2 bu. potatoes, @ 65¢	1	30
"	17	1 can baking powder		40
		Received payment, Oct. 5, 1900. J. H. EDWARDS.	$6	15

Explanation: (1) Mr. Edwards, the merchant, makes the bill for his customer, Mr. Adams. (2) Mr. Edwards lives at San Francisco, and made the bill Oct. 3, 1900. (3) Mr. Adams lives at Oakland. (4) "Dr." means debtor—Mr. Adams is debtor *to Mr. Edwards.* (5) The sign, %, stands for *account;* and "@" stands for *at.* (6) The columns on the left are for dates; those on the right for dollars and cents. (7) The bill was paid Oct. 5, 1900, as shown by Mr. Edwards's receipt on the bill.

People living in or near towns or cities usually keep their money in a bank, and pay their bills by giving **checks** on the bank.

[A CHECK.]

FIRST NATIONAL BANK OF SAN FRANCISCO, CALIFORNIA.

OAKLAND, CALIFORNIA, Oct. 5, 1900.

Pay to J. H. EDWARDS, or order, Six & $\frac{15}{100}$ DOLLARS ($6.15).

H. G. ADAMS.

Explanation: This check shows that Mr. Adams, who lives at Oakland, did, on Oct. 5, 1900, direct the First National Bank of San Francisco to pay Mr. Edwards the amount stated in the check.

EXERCISE CXIV.

Selecting your own names and dates, make out the following bills of goods, receipt the bills, and draw checks for their payment:

1.
12 yd. muslin, @ 8⅓¢.
15 yd. sheeting, @ 30¢.
2 pairs shoes, @ $2.40.
10 yd. silk, @ $1.30.
9 yd. calico, @ 8¢.
20 yd. ribbon, @ 10¢.

2.
3 plows, @ $8.75.
2 harrows, @ $15.
1 binder, $125.
1 hay rake, $45.
1 buggy, $75.
1 wagon, $65.

3.
7 lb. butter, @ 15¢.
5 doz. eggs, @ 10¢.
8 lb. sugar, @ 12½¢.
2 lb. coffee, @ 25¢.
5 bars soap, @ 5¢.
1 pk. sweet potatoes, 20¢.

4.
10 lb. pork, @ 10¢.
6 lb. ham, @ 12½¢.
12 lb. beef, @ 7¢.
5 lb. lard, @ 11¢.
9 lb. mutton, @ 8⅓¢.
4 lb. sausage, @ 10¢.

5.
2 sacks bran, @ 75¢.
10 bu. apples, @ 90¢.
9 bu. peaches, @ 80¢.
30 lb. dried apples, @ 5¢.
5 sacks flour, @ $1.75.
13 cans pears, @ 20¢.

6.
1 horse, $95.
12 cows, @ $30.
15 sheep, @ $3.50.
11 calves, @ $7.50.
2 mules, @ $120.
13 pigs, @ $2.50.

123. Odd Problems in Fractions.

EXAMPLES.

1. What number added to $\frac{2}{5}$ of itself will make 35?

Solution: (1) $\frac{5}{5}$ of No. + $\frac{2}{5}$ of No. = 35. (Basis.)
(1) = (2) $\frac{7}{5}$ of No. = 35.
$\frac{1}{7}$ of (2) = (3) $\frac{1}{5}$ of No. = 5.
5 × (3) = (4) $\frac{5}{5}$ of No. = 25, answer.

2. What number less its $\frac{3}{8}$ leaves 50?

Solution: (1) $\frac{8}{8}$ of No. $-\frac{3}{8}$ of No. $=50$. (Basis.)
(1) $=$ (2) $\frac{5}{8}$ of No. $=50$.
$\frac{1}{8}$ of (2) $=$ (3) $\frac{1}{8}$ of No. $=10$.
$8\times$(3) $=$ (4) $\frac{8}{8}$ of No. $=80$, answer.

3. Find that number whose $\frac{3}{7}$ added to its $\frac{4}{5}$ will make 129.

Basis: $\frac{3}{7}$ of No. $+\frac{4}{5}$ of No. $=129$.

4. Find that number whose $\frac{5}{6}$ less its $\frac{3}{8}$ leaves 22.

Basis: $\frac{5}{6}$ of No. $-\frac{3}{8}$ of No. $=22$.

5. 96 is $\frac{1}{2}$ more than what number?

Solution: (1) The No. $+\frac{1}{2}$ of No. $=96$. (Basis.)
(1) $=$ (2) $\frac{3}{2}$ of No. $=96$.
$\frac{1}{2}$ of (2) $=$ (3) $\frac{1}{2}$ of No. $=32$.
$2\times$(3) $=$ (4) $\frac{2}{2}$ of No. $=64$, answer.

6. 32 is $\frac{1}{3}$ less than what number?

Solution: (1) The No. $-\frac{1}{3}$ of No. $=32$. (Basis.)
(1) $=$ (2) $\frac{2}{3}$ of No. $=32$.
$\frac{1}{3}$ of (2) $=$ (3) $\frac{1}{3}$ of No. $=16$.
$3\times$(3) $=$ (4) $\frac{3}{3}$ of No. $=48$, answer.

The product of two factors divided by one of the factors will give the other factor. (Commit.) To illustrate:

$$5\times4=20.$$

Then, $20\div5=4$.

Or, $20\div4=5$.

7. What number multiplied by $\frac{3}{5}$ will give 33 for a quotient?

Question: (1) $\frac{3}{5}\times(\ \ )=33$?

Answer: (2) $33\div\frac{3}{5}=\frac{11}{\cancel{33}}\times\frac{5}{\cancel{3}}=55$.

Explanation: This example gives one factor ($\frac{3}{5}$) and a product (33), to find the other factor.

The dividend is equal to the product of the divisor and quotient; the divisor is equal to the dividend divided by the quotient. (Commit.)

To illustrate: $20 \div 5 = 4$.
Then, $4 \times 5 = 20$.
And $20 \div 4 = 5$.

8. By what must 12 be divided to give a quotient of 26?

Question: (1) $12 \div (\quad) = 26$?
Answer: (2) $12 \div 26 = \frac{12}{26} = \frac{6}{13}$.

9. What number divided by $\frac{3}{10}$ will give 540 for a quotient?

Question: (1) $(\quad) \div \frac{3}{10} = 540$?
Answer: (2) $\frac{3}{10} \times 540 = 162$.

10. 12 is what part of 42?

Solution: (1) $1 = \frac{1}{42}$ of 42. (Basis.)
$12 \times (1) = (2)$ $12 = \frac{12}{42}$ or $\frac{2}{7}$ of 42, answer.

11. 240 is how many times 60?

Solution: (1) $1 = \frac{1}{60}$ of 60. (Basis.)
$240 \times (1) = (2)$ $240 = \frac{240}{60}$ of 60, or 4×60, answer.

EXERCISE CXV.

1. What number added to its $\frac{5}{4}$ will give 228?

2. What number added to $3\frac{1}{2}$ times itself will give 221?

3. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ of a number added together make 135. Find the number.

4. 275 is $\frac{1}{4}$ more than what number?

5. Mr. Jones, who is 63 years old, is $\frac{3}{4}$ older than his son. How old is the son?

6. What number less its $\frac{5}{8}$ leaves a remainder of 364?

7. What number is 56 more than its $\frac{9}{7}$?

8. After depositing $\frac{5}{2}$ of my money in the bank, I found that I had \$441 left. How much money had I altogether?

9. Mr. Brown bought $\frac{5}{7}$ of a boat. If the remaining part is worth \$4500, at that price what is the whole boat worth, and what did Mr. Brown pay for his part?

10. $\frac{5}{9}$ of a number less $\frac{2}{7}$ of it leaves 68. Find the number.

11. 275 is $\frac{3}{8}$ less than what number?

12. If I had $\frac{1}{5}$ more money than I now have, I would have $264. How much money have I?

13. $\frac{8}{11} \times (\quad) = 56$?

14. What number multiplied by $\frac{3}{5}$ will give 256 for a product?

15. I sold $\frac{5}{18}$ of my crop for $250. At that price, what is the whole crop worth?

Value of $\frac{5}{18}$ crop = $250. (Basis.)

16. $\frac{7}{18} \div (\quad) = \frac{7}{8}$?

17. $(\quad) \div \frac{5}{13} = 19\frac{1}{2}$?

18. What number divided by $\frac{8}{21}$ will give 630 for a quotient?

19. $\frac{7}{15}$ divided by what number will give $\frac{14}{25}$ for a quotient?

20. A man, 36 years old, is $\frac{2}{5}$ younger than his father. How old is the father?

21. 75 is what part of 125?

22. I deposit $350 and keep $450. What part of the whole have I in bank?

X. DECIMAL FRACTIONS.

124. Definitions.—A Decimal Fraction has for its expressed denominator 1 with 0's annexed. Thus:

$$\frac{1}{10},\ \frac{5}{100},\ \frac{124}{10000},\ \frac{2045}{100000}.$$

A decimal fraction may be written without using figures for its denominator. Thus:

$$\frac{1}{10} = .1$$
$$\frac{5}{100} = .05$$
$$\frac{124}{10000} = .124$$
$$\frac{2045}{100000} = .2045$$

By this method of writing decimal fractions, (1) the numerator is written just as an integer, (2) a period, called a **Decimal Point,** is put on the left of as many places in the

numerator as there are 0's in the expressed denominator. If there are not so many places in the numerator as are needed, 0's are prefixed to the numerator to make the required number of places.

A decimal fraction written in this way is called a **Decimal.**

The denominator of a decimal—

of one place is 10; .7 is read $\frac{7}{10}$;
of two places is 100; .46 is read $\frac{46}{100}$;
of three places is 1000; .307 is read $\frac{307}{1000}$;
of four places is 10000; .0453 is read $\frac{453}{10000}$; and so on.

The decimal places are named from the decimal point to the right, as follows:

The first place, **tenths;**
the second place, **hundredths;**
the third place, **thousandths;**
the fourth place, **ten-thousandths;**
the fifth place, **hundred-thousandths;** and so on.

The denominator of a decimal is indicated by the name of its right-hand place. Thus, in

.0246

the name of the right-hand place is *ten-thousandths;* the denominator is *ten thousand.*

When a decimal is annexed to an integer the expression is called a **Mixed Decimal.** Thus:

5.04; and is read, $5\frac{4}{100}$.

A common fraction may be annexed to a decimal; such an expression is called a **Complex Decimal.** Thus:

.12½; and is read, $\frac{12\frac{1}{2}}{100}$.

5.37½; and is read, $5\frac{37\frac{1}{2}}{100}$.

EXERCISE CXVI.

Read:

1. .8	*7.* .001	*13.* 10.0001
2. .18	*8.* .246	*14.* .04567
3. .05	*9.* 1.024	*15.* $9.62\frac{1}{2}$
4. .86	*10.* .0345	*16.* $15.25\frac{3}{4}$
5. 5.3	*11.* .9846	*17.* $.035\frac{16}{25}$
6. .021	*12.* 3.0201	*18.* $34.062\frac{1}{2}$

Write as Decimals:

19. $\frac{3}{10}$	*27.* $\frac{346}{1000}$	*35.* $12\frac{62\frac{1}{2}}{100}$
20. $\frac{3\frac{1}{2}}{10}$	*28.* $\frac{12}{100000}$	*36.* $\frac{346}{10000}$
21. $\frac{5}{10}$	*29.* $\frac{201}{10000}$	*37.* $\frac{8640}{100000}$
22. $\frac{17}{100}$	*30.* $5\frac{18}{100}$	*38.* $\frac{100001}{1000000}$
23. $\frac{17}{1000}$	*31.* $7\frac{9}{1000}$	*39.* $100\frac{1}{10000}$
24. $\frac{84}{100}$	*32.* $15\frac{24}{1000}$	*40.* $75\frac{1\frac{1}{4}}{1000}$
25. $\frac{54\frac{1}{2}}{100}$	*33.* $17\frac{35\frac{1}{8}}{100}$	*41.* $31\frac{808}{10000}$
26. $\frac{42\frac{1}{2}}{1000}$	*34.* $\frac{33\frac{1}{3}}{100}$	*42.* $11\frac{1111\frac{1}{111}}{100000}$

125. Reducing Decimals to Higher Terms.— If a 0 is annexed to a decimal, the numerator is multiplied by 10; but since another decimal place is thus added, the denominator is also multiplied by 10, and the value of the fraction is not changed. Thus:

$$.5 = .50 = .500 = .5000, \text{ etc.}$$

That is, $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000} = \frac{5000}{10000}$, etc.

EXAMPLE.

Reduce .125 to 100000ths.

Process: .125 = .12500, result.

Explanation: Annex 0's to the decimal till the number of decimal places is equal to the number of 0's in the required denominator.

EXERCISE CXVII.

Reduce —

1. .2 to 100ths.
2. .7 to 1000ths.
3. .12 to 1000ths.
4. 7.5 to 100ths.
5. .08 to 1000ths.
6. 17.42 to 10000ths.
7. 84.5 to 1000ths.
8. .72 to 100000ths.
9. .1 to 10000ths.
10. 3.142 to 100000ths.

126. Reducing Decimals to Lower Terms.—If there are 0's at the right of a decimal, they may be dropped without changing the value of the decimal. Thus:

$.2300 = .23$; that is,

$\frac{2300}{10000} = \frac{23}{100}$.

EXAMPLES.

1. Reduce .750 to 100ths.

Process: .750 = .75, result.

2. Reduce .24000 to 1000ths.

Process: .24000 = .240, result.

EXERCISE CXVIII.

Reduce—

1. .740 to 100ths.
2. .8500 to 100ths.
3. .5000 to 10ths.
4. .11000 to 1000ths.
5. .30000 to 10ths.
6. 8.5000 to 100ths.
7. 14.31000 to 100ths.
8. 10.8000 to 10ths.
9. 91.1000 to 100ths.
10. 84.7050 to 1000ths.

127. Addition of Decimals.—Write the numbers to be added so that the decimal points will form a column. *Add as in integers, placing decimal point in the result beneath the column of decimal points above.*

EXAMPLE.

1. Add .501, 3.45, .125, 7.0034, 18.3024.

Long Way.	*Explanation:* In the *long way*, all decimals are reduced to the highest denomination, 10000ths. The denomination of the sum is 10000ths. Why?	*Short Way.*
.5010		.501
3.4500		3.45
.1250		.125
7.0034		7.0034
18.3024		18.3024
29.3818, result.		29.3818, result.

In the *short way*, the decimals are not reduced to higher denominations. The 0's on the right do not affect the result. Use the *short way* in practice.

EXERCISE CXIX.

Add:

1. .701, .425, .846, .972.
2. 3.04, 5.5, 71.34, 19.43, 10.01.
3. 8.0034, 170.25, 34.842, 75.6004.
4. 99.999, 77.707, 60.0006, 19.8088.
5. 136.003, 700.07, 504.371, 703, 268.
6. $12.25, $15.80, $9.05, $74.50.

NOTE.—When an expression of $'s has only two decimal places, call the decimal part *cents;* if there are more than two decimal places, read the decimal part as a fraction of a dollar.

7. $425, $561.65, $842.4756, $92.0005.
8. 1000, 5.00001, 16.037, 100,0001.
9. 1000.0001, 99.999, .0607, .00079.
10. 888.88888, 76.5004, 99.879, .898765.

128. Subtraction of Decimals.—Write the numbers so that the decimal point in the subtrahend will be beneath the decimal point in the minuend. *Subtract as in integers, placing the decimal point in the result beneath decimal points above.*

EXAMPLES.

1. From 5.47 take 2.638.

Process.

5.470
2.638
2.832, result.

Explanation: Reduce the minuend to the denomination of the subtrahend by annexing a 0 to it. The denomination of the result is 1000ths. Why?

2. From .54652 take .369.

Long Way.

.54652
.36900
.17752, result.

Explanation: In the *long way*, the subtrahend is reduced to the denomination of the minuend. The denomination of the result is 10000ths. Why? In the *short way*, the subtrahend is not reduced to higher denomination. The 0's on the right do not affect the result. Use the *short way* in practice.

Short Way.

.54652
.369
.17752, result.

EXERCISE CXX.

1. From 5.034 take 3.521.
2. From 1.00005 take .34792.
3. From .0346 take .0099.
4. From 3.846 take .9999.
5. From 34.1 take .256.
6. From 25 take 2.34567.
7. From 5.00006 take .375.
8. From $34.25 take $8.80.
9. From $1 take $.25.
10. From $3 take 10¢.

Express the 10¢ with "$."

11. From $3.20 take $.9865.
12. From $50.04 take $4.653.

129. Multiplication of Decimals.—*Multiply as in integers, neglecting ciphers that may be on the left of the multiplicand and multiplier, and place the decimal point in the result so that there will be as many decimal places in the product as are in both multiplicand and multiplier*

EXAMPLES.

1. Multiply .852 by .35.

Process.

.852
.35
4260
2556
.29820, result.

Explanation: The multiplicand is 852 *thousandths;* the multiplier is 35 *hundredths.* Hundredths times thousandths gives hundred-thousandths. Show this by use of common fractions. Hundred-thousandths is expressed by a decimal of 5 places. After placing the decimal point, cancel the 0 on the right, for the product should be expressed in its lower terms.

2. Multiply .0037 by 2.5.

Process.

.0037
2.5
185
74
.00925, result.

Explanation: For the purpose of the multiplication, the multiplicand is 37 (neglecting the 0's on the left).

The product after multiplying contains but *three* figures; but there must be *five* decimal places. Why? Therefore two 0's are prefixed and the decimal point is placed to the left.

3. Multiply 246 by 3.07.

Process.

246
3.07
1722
738
755.22

NOTE.—Have pupil explain.

EXERCISE CXXI.

1. Multiply .247 by .231.
2. Multiply .059 by .128.
3. Multiply .394 by .0257.
4. Multiply 3.41 by 125.
5. Multiply 249 by .0062.
6. Multiply 7.304 by 92.4.
7. Multiply 10.001 by .01.
8. Multiply .0001 by .001.
9. Multiply 1000 by .0001.
10. Multiply $121.076 by 20.08.
11. Multiply $12.55 by 3.77.
12. Multiply $1000 by .347.

130. Division of Decimals.—*Divide as in integers, neglecting 0's that may be on the left of the dividend or divisor.*

Since the dividend is equal to the product of the divisor and quotient (see p. 181), the dividend will have as many decimal

places as both divisor and quotient (see Art. 129). From this, the following conclusions are evident:

(1) *The dividend must not have fewer decimal places than the divisor.* (Commit.)

NOTE.—If the dividend has not as many decimal places as the divisor, enough 0's to make them equal should be annexed to the dividend before dividing.

EXAMPLE.

Prepare to divide .15 by .0025.

Form.

.0025).1500

Explanation: Two 0's must be annexed to the dividend, because there are four decimal places in the divisor and only two decimal places in the dividend.

(2) *If, after dividing, the number of decimal places used in the dividend equals the number in the divisor, the quotient is an integer.* (Commit.)

EXAMPLE.

Divide .15 by .0025.

Process.

60, result.
.0025).1500
150
0

NOTE.—In the actual process of division, the two 0's on the left of the divisor are not used; the divisor is 25.

(3) *If, after dividing, the number of decimal places used in the dividend exceeds the number of decimal places in the divisor, the number of decimal places in the quotient must equal that excess.* (Commit.)

EXAMPLES.

1. Divide .98745 by .29.

3.4 0 5, result.
.2 9).9 8 7 4 5
8 7
1 1 7
1 1 6
1 4 5
1 4 5

Explanation: There are *five* decimal places in the dividend and *two* in the divisor. The *excess* is three. Therefore, the quotient must contain *three decimal places.*

2. Divide .002868 by .239.

Process.

.012, result.
.239).002868
239
478
478

Explanation: After dividing the 2868 (neglecting the 0's on the left) by 239, the quotient is 12; but the *excess* of decimal places in the dividend over those in the divisor is *three*. Therefore, the quotient must contain *three decimal places*.

If there is a remainder after using the last figure of the dividend, 0's may be annexed to the dividend and the division continued at pleasure.

EXAMPLE.

Divide 2.09 by .017.

Process.

122.94117+, result.
.017)2.09000000
17
39
34
50
34
160
153
70
68
20
17
30
17
130
119
11

Explanation: The plus sign after the quotient shows that there is a remainder, and that the division could be continued further if desired. After dividing as far as desired, locate the decimal point according to Rule 3.

EXERCISE CXXII.

1. Divide 1.25 by .5.
2. Divide .325 by 1.3.
3. Divide .0405 by .0045.
4. Divide 3.402 by 1.8.
5. Divide .0064 by 40.
6. Divide 4.50 by 1.28.
7. Divide 85.51 by .017.
8. Divide 1000 by .001.
9. Divide .0001 by 10000.
10. Divide 16 by 64.
11. Divide $78.52 by $2.60.
12. Divide $16.32 by $.064.

131. Reducing Common Fractions to Decimals.—Since a common fraction is an expression of division, if the division be performed the result will be an integer or decimal. In dividing, observe the rules for division of decimals.

EXAMPLES.

1. Reduce $\frac{3}{16}$ to a decimal.

Process.

.1875, result.
16)3.0000
16
140
128
120
112
80
80

2. Reduce $\frac{2}{7}$ to a decimal (continued 6 places).

7)2.000000
.285714+, result.

NOTE.—There will always be a remainder if the divisor contains any factor other than 2 or 5.

3. Reduce $9\frac{5}{17}$ to a mixed decimal (continued 4 places).

Process.

.2941+
17)5.0000
34
160
153
70
68
20
17
3

Result: 9.2941+.

NOTE.—Reduce the fractional part to a decimal and annex the decimal to the integer.

EXERCISE CXXIII.

Reduce to decimals, obtaining complete results:

1. $\frac{3}{40}$
2. $\frac{5}{80}$
3. $\frac{18}{20}$
4. $5\frac{7}{8}$
5. $\frac{11}{16}$
6. $\frac{43}{50}$
7. $\frac{9}{32}$
8. $12\frac{7}{16}$
9. $14\frac{9}{40}$
10. $15\frac{17}{64}$
11. $\frac{111}{320}$
12. $11\frac{9}{80}$

Obtain results true to 5 places:

13. $\frac{5}{12}$	*17.* $7\frac{11}{18}$	*21.* $\frac{83}{90}$
14. $\frac{8}{15}$	*18.* $10\frac{5}{7}$	*22.* $7\frac{5}{19}$
15. $\frac{1}{9}$	*19.* $\frac{17}{35}$	*23.* $\frac{1}{21}$
16. $5\frac{5}{6}$	*20.* $\frac{53}{86}$	*24.* $18\frac{8}{11}$

132. Reducing Decimals to Common Fractions.—Express the denominator, which is 1 with as many 0's annexed as there are decimal places. Then reduce the result to its simplest form.

EXAMPLES.

1. Reduce .15 to a common fraction.

Process: $.15 = \frac{15}{100} = \frac{3}{20}$, result.

2. Reduce .0036 to a common fraction.

Process: $.0036 = \frac{36}{10000} = \frac{9}{2500}$, result.

3. Reduce 5.125 to a mixed number.

NOTE.—Reduce the decimal part to a common fraction and annex it to the integer.

Process: $.125 = \frac{125}{1000} = \frac{1}{8}$; $\therefore$ the result is $5\frac{1}{8}$.

4. Reduce $.12\frac{1}{2}$ to a common fraction.

Process: $.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{1}{8}$, result.
4

EXERCISE CXXIV.

Reduce to a common fraction or mixed number:

1. .125	*5.* .016	*9.* $.44\frac{4}{9}$
2. .625	*6.* .0075	*10.* $5.37\frac{1}{2}$
3. .072	*7.* $.16\frac{2}{3}$	*11.* $7.06\frac{1}{4}$
4. 7.375	*8.* $.33\frac{1}{3}$	*12.* $15.66\frac{2}{3}$

133. Reciprocals.—The **Reciprocal** of a number is one divided by that number.

EXAMPLES.

1. The reciprocal of $5 = \frac{1}{5}$.
2. The reciprocal of $\frac{5}{9} = 1 \div \frac{5}{9} = 1 \times \frac{9}{5} = \frac{9}{5}$.

NOTE.—The reciprocal of a common fraction is that fraction inverted. (See Article 90.)

3. The reciprocal of $\frac{7}{2} = \frac{2}{7}$.
4. The reciprocal of $3\frac{1}{2} = 1 \div 3\frac{1}{2} = 1 \times \frac{2}{7} = \frac{2}{7}$.

NOTE.—Would you expect the reciprocal of $\frac{7}{4}$ to be the same as that of $3\frac{1}{2}$? Why?

5. The reciprocal of $7\frac{2}{9} = \frac{9}{65}$.
6. The reciprocal of $.12 = \frac{1}{.12}$.
7. The reciprocal of $4.25 = \frac{1}{4.25}$

EXERCISE CXXV.

Give orally the reciprocal of each of the following:

1. 5	6. $12\frac{3}{4}$	11. $\frac{72}{18}$
2. $\frac{3}{11}$	7. .25	12. $8\frac{5}{7}$
3. $\frac{1}{12}$	8. 2.4	13. $10\frac{1}{10}$
4. 5.25	9. $\frac{34}{42}$	14. .342
5. $8\frac{1}{2}$	10. $\frac{18}{29}$	15. 17

134. The Use of Reciprocals.—After stating the basis of a problem, often one of the requirements is to go from what is given to unity. (See Article 26.) *This may always be done by multiplying by the reciprocal of the number in the first member of the basis.*

EXAMPLES.

1. If 20 books cost \$50, what cost 1 book?

Solution: (1) Cost of 20 books = \$50. (Basis.)
$\frac{1}{20} \times (1) = (2)$ Cost of 1 book = \$2.50, answer.

NOTE.—Did we multiply by the reciprocal of 20? We have heretofore used "of" instead of "×"; is "of" correct? (See Article 88.)

2. If $3\frac{1}{3}$ cords of wood are worth \$8.00, what is 1 cord worth?

Solution: (1) Price of $3\frac{1}{3}$ cords = \$8.00. (Basis.)
$\frac{3}{10}$ of (1) = (2) Price of 1 cord = \$2.40, answer.

NOTE.—$\frac{3}{10}$ is the reciprocal of $3\frac{1}{3}$.

3. At \$1.25 per bu., how much wheat can be bought for \$22.50?

Solution: (1) \$1.25=cost of 1 bu. (Basis.)
$\frac{1}{1.25} \times (1) = (2)$ \$1 = cost of $\frac{1}{1.25}$ bu.
$22.50 \times (2) = (3)$ \$22.50 = cost of $\frac{22.50}{1.25}$, or 18 bu., answer.

NOTE.—$\frac{1}{1.25}$ is the reciprocal of 1.25.

4. $12\frac{1}{2}$ is what part of $37\frac{1}{2}$?

Solution: (1) $12\frac{1}{2}$ = ( ) of $37\frac{1}{2}$? (Question.)
(2) $37\frac{1}{2}$ = all of $37\frac{1}{2}$. (Basis.)
$\frac{2}{75}$ of (2) = (3) 1 = $\frac{2}{75}$ of $37\frac{1}{2}$.
$12\frac{1}{2} \times (3) = (4)$ $12\frac{1}{2} = \frac{25}{2} \times \frac{2}{75}$ or $\frac{1}{3}$ of $37\frac{1}{2}$, answer.
3

NOTE.—$\frac{2}{75}$ is the reciprocal of $37\frac{1}{2}$.

EXERCISE CXXVI.

1. If 25 horses cost \$1631.25, what is the average cost per horse?

2. If 12.5 bu. of apples are worth \$10.625, what are 18.2 bu. worth?

3. In $21\frac{3}{4}$ days, a man earns \$32.625. How much is that per day?

4. $\frac{5}{12}$ of my money is \$8.40. How much money have I?

5. $37\frac{1}{2}$ is what part of $87\frac{1}{2}$?

6. \$5.40 is what part of \$12.60?

135. Parts of a Dollar.—The following equivalent fractions of a dollar are so much used in business that they should be committed to memory by the pupil:

EQUIVALENTS.

$.06¼ = $1/16
$.08⅓ = $1/12
$.10 = $1/10
$.12½ = $1/8
$.16⅔ = $1/6
$.20 = $1/5
$.25 = $1/4
$.33⅓ = $1/3
$.50 = $1/2

NOTE. The common fractions are usually more convenient than the decimals

EXAMPLES.

1. Find the value of 24 books, at $.33⅓ each.

Oral Solution: Since 1 book is worth $.33⅓ or $1/3, 24 books are worth $24/3, or $8, answer.

2. At 16⅔¢ each, how many knives can be bought for $3.50?

Oral Solution: Since $1/6 buys 1 knife, $1 will buy 6 knives, and $3½ will buy 3½ times 6 knives, or 21 knives, answer.

EXERCISE CXXVII.

Solve orally:

1. What will 218 dozen eggs sell for, at 12½¢ per dozen?
2. What will 50 lb. of butter cost, at 20¢ per pound?
3. What are 80 knives worth, at 25¢ each?
4. I bought 15 bu. of corn, at 33⅓¢ per bu. How much did it cost me?
5. What will 64 yds. of cloth cost, at 6¼¢ per yd.?
6. I bought 24 quarts of strawberries, at 8⅓¢ per qt. How much did I pay for them?

7. At $12\frac{1}{2}¢$ each, how many tin pails can be bought for $4?

8. At $25¢$ each, how many melons can be bought for $5?

9. At $50¢$ per bu., how many bushels of apples can be bought for $12?

10. At $33\frac{1}{3}¢$ each, how many books can be bought for $11?

136. Reduction of Decimal Denominate Numbers.—Express all common fractions as decimals, and proceed as on page 137.

EXAMPLES.

1. Reduce .342 bu. to qt.

Solution: (1) 1 bu. = 4 pk. (Basis.)
.342 × (1) = (2) .342 bu. = 1.368 pk.
(3) 1 pk. = 8 qt. (Basis.)
1.368 × (3) = (4) 1.368 pk. = 10.944 qt., result.

2. Reduce .3125 bu. to lower integral units.

Solution: (1) 1 bu. = 4 pk. (Basis.)
.3125 × (1) = (2) .3125 bu. = 1.25 pk.

NOTE.—Leaving the 1 pk., reduce the .25 to qt.

(3) 1 pk. = 8 qt. (Basis.)
.25 × (3) = (4) .25 pk. = 2 qt.
∴ .3125 bu. = 1 pk. 2 qt.

3. Reduce .12 lb. to the decimal of a ton.

Solution: (1) 1 lb. = .01 cwt. (Basis.)
.12 × (1) = (2) .12 lb. = .0012 cwt.
(3) 1 cwt. = $\frac{1}{20}$ T. = .05 T. (Basis.)
.0012 × (3) = (4) .0012 cwt. = .00006 T., result.

4. Reduce 3 qt. 1 pt. to the decimal of a bu.

Solution: (1) 1 pt. = $\frac{1}{2}$ qt. = .5 qt. (Basis.)
(2) 3 qt. + .5 qt. = 3.5 qt.
(3) 1 qt. = $\frac{1}{8}$ pk. = .125 pk. (Basis.)
3.5 × (3) = (4) 3.5 qt. = .4375 pk.
(5) 1 pk. = $\frac{1}{4}$ bu. = .25 bu. (Basis.)
.4375 × (5) = (6) .4375 pk. = .109375 bu., result.

EXERCISE CXXVIII.

1. Reduce .025 yd. to the decimal of an inch.
2. Reduce .25 bu. to pints.
3. Reduce .0346 cwt. to pounds.
4. Reduce .8 lb. troy to lower integral units.
5. Reduce .346 T. to lower integral units.
6. Reduce .025 yr. to lower integral units.
7. Reduce 125 links to the decimal of a mile.
8. Reduce 10 ℈ to the decimal of a lb.
9. Reduce .32 rd. to the decimal of a mile.
10. Reduce 2 pk. 5 qt. to the decimal of a bu.
11. What are 3 pk. of potatoes worth, at $.80 per bushel?
12. What will 5 T. 3 cwt. of coal bring, at $3.50 per ton?

XI. PERCENTAGE.

137. Explanation of the Term Per Cent.—In arithmetic, the term **Per Cent** means **hundredth** or **hundredths.** The sign, %, is used for the words *per cent.*

$$1\% = \tfrac{1}{100};$$
$$5\% = \tfrac{5}{100};$$
$$12\tfrac{1}{2}\% = \tfrac{12\frac{1}{2}}{100};\text{ and}$$
$$100\% = \tfrac{100}{100}.$$

You should not think of percentage as something new in principle; it is only a new form of expressing *hundredths.*

Remember that 100% of a number is all of it; or that all of a number is 100% of it.

138. To Find Some Number of Per Cent of a Given Number.—When a number is given, *100% of it is given.* By aid of this fact, we will form the *Basis* in each of the following problems.

Many of the per cents that we will need to find are simple fractions of 100%. To aid you in learning these, I have given the following table:

TABLE.

$$2\% = \tfrac{1}{50} \text{ of } 100\%$$
$$4\% = \tfrac{1}{25} \text{ of } 100\%$$
$$5\% = \tfrac{1}{20} \text{ of } 100\%$$
$$6\tfrac{1}{4}\% = \tfrac{1}{16} \text{ of } 100\%$$
$$8\tfrac{1}{3}\% = \tfrac{1}{12} \text{ of } 100\%$$
$$10\% = \tfrac{1}{10} \text{ of } 100\%$$
$$12\tfrac{1}{2}\% = \tfrac{1}{8} \text{ of } 100\%$$
$$16\tfrac{2}{3}\% = \tfrac{1}{6} \text{ of } 100\%$$
$$20\% = \tfrac{1}{5} \text{ of } 100\%$$
$$25\% = \tfrac{1}{4} \text{ of } 100\%$$
$$33\tfrac{1}{3}\% = \tfrac{1}{3} \text{ of } 100\%$$
$$50\% = \tfrac{1}{2} \text{ of } 100\%$$

EXAMPLES.

1. Find 12% of 630 pounds.

Solution: (1) 12% of 630 lb. = ( ) lb.? (Question.)
(2) 100% of 630 lb. = 630 lb. (Basis.)
$\frac{1}{100}$ of (2) = (3) 1% of 630 lb. = 6.3 lb.
12 × (3) = (4) 12% of 630 lb. = 75.6 lb., ans.

NOTE.—In this problem, we went from 100% to 1%; then from 1% to 12%.

2. Mr. Jones has $1600, of which 25% is in the bank. How much has he in the bank?

Solution: (1) 100% of $1600 = $1600. (Basis.)
$\frac{1}{4}$ of (1) = (2) 25% of $1600 = $400, ans.

NOTE.—By knowing (from the table) that 25% is $\frac{1}{4}$ of 100%, we at once take $\frac{1}{4}$ of the Basis, which gives the required answer without having to pass first to unity.

3. I sell goods for Mr. Carr, amounting to $400, on which I charge a commission of 2%. Find my commission.

Solution: (1) 100% of sales = $400. (Basis.)
$\frac{1}{50}$ of (1) = (2) 2% of sales = $8, ans.

NOTE.—Commission for selling is always some % of the selling price.

4. I bought goods for a merchant, amounting to $80. I charged 5% commission. Find my commission.

Solution: (1) 100% of purchase = $80. (Basis.)
$\frac{1}{20}$ of (1) = (2) 5% of purchase = $4, ans.

NOTE.—Commission for buying is always some % of the purchase price.

5. An article cost $4.20, and was sold at a profit of $16\frac{2}{3}\%$. Find the gain.

Solution: (1) 100% of cost = $4.20. (Basis.)
$\frac{1}{6}$ of (1) = (2) $16\frac{2}{3}\%$ of cost = $.70, ans.

NOTE.—The profit or loss is always some % of the cost price.

6. I bought goods for $300, and sold them at $33\frac{1}{3}\%$ profit. Find the selling price.

Solution: (1) 100% of cost = $300. (Basis.)
$\frac{1}{3}$ of (1) = (2) $33\frac{1}{3}\%$ of cost = $100.
(3) $300 + $100 = $400, ans.

NOTE.—The *profit* added to the *cost* gives the *selling* price.

7. I bought a horse for $72, and sold him at a loss of $12\frac{1}{2}\%$. Find the selling price.

Solution: (1) 100% of cost = $72. (Basis.)
$\frac{1}{8}$ of (1) = (2) $12\frac{1}{2}\%$ of cost = $9.
(3) $72 − $9 = $63, ans.

NOTE.—The *loss* subtracted from the *cost* gives the selling price.

EXERCISE CXXIX.

1. Find 25% of $440.

2. Find 20% of $3400.

3. Of a flock of 480 sheep, 16⅔% were lambs. How many lambs were there? How many sheep besides the lambs?

4. A man owes me $64, and pays 25% of the debt. How much does he pay? How much does he still owe?

5. A commission merchant sells for me 50 head of hogs, averaging 250 pounds each, at 4¢ per lb. He charges 2% commission for selling. Find the amount of his commission.

6. A lawyer charges me 5% for collecting a debt of $700. How much does the lawyer get? How much do I get?

7. What is the selling price, if an article bought for $3.80 sells at a 25% profit?

8. An article that cost $5 sells at a 20% loss. Find the selling price.

139. To Find a Number, when Some Number of Per Cent of it is Given.—In the following problems, we always want all of the number, or *100% of it.*

The following table will aid in shortening the solutions:

TABLE.

$$100\% = 50 \times 2\%$$
$$100\% = 25 \times 4\%$$
$$100\% = 20 \times 5\%$$
$$100\% = 16 \times 6\tfrac{1}{4}\%$$
$$100\% = 12 \times 8\tfrac{1}{3}\%$$
$$100\% = 10 \times 10\%$$
$$100\% = 8 \times 12\tfrac{1}{2}\%$$
$$100\% = 6 \times 16\tfrac{2}{3}\%$$
$$100\% = 5 \times 20\%$$
$$100\% = 4 \times 25\%$$
$$100\% = 3 \times 33\tfrac{1}{3}\%$$
$$100\% = 2 \times 50\%$$

1. 17% of a number is 68. Find the number.

Solution: (1) 17% of number = 68. (Basis.)
$\frac{1}{17}$ of (1) = (2) 1% of number = 4.
100 × (2) = (3) 100% of number = 400, ans.

NOTE.—When we cannot solve by a shorter plan, we can always go from what we have given to 1%; then from 1% to the required number of %.

2. A lawyer is paid 5% for collecting a debt. If he gets $45, what is the amount of the debt?

Solution: (1) 5% of debt = $45. (Basis.)
20 × (1) = (2) 100% of debt = $900, ans.

3. My gain on a certain article, which I sold at a profit of 25%, is $.75. Find the cost price.

Solution: (1) 25% of cost = $.75. (Basis.)
4 × (1) = (2) 100% of cost = $3, ans.

4. My loss on a certain article, which I sold at a loss of $16\frac{2}{3}$%, is $1.25. Find the cost price.

Solution: (1) $16\frac{2}{3}$% of cost = $1.25. (Basis.)
6 × (1) = (2) 100% of cost = $7.50, ans.

5. In one book case I have 50 books; this is $12\frac{1}{2}$% of my library. How many books have I in my library?

Solution: (1) $12\frac{1}{2}$% of library = 50 books. (Basis.)
8 × (1) = (2) 100% of library = 400 books, ans.

EXERCISE CXXX.

1. $60 is 20% of what amount?

2. 180 books is $16\frac{2}{3}$% of what number of books?

3. At 2% commission, a commission merchant receives $8 for selling 40 barrels of molasses. For how much per barrel did the molasses sell?

4. A lawyer charges 3% commission for collecting a debt. His commission amounts to $60; find the amount of the debt.

5. Tea sold at $6\frac{1}{4}\%$ profit, yields a gain of $5¢$ per pound. Find the cost price per pound.

6. 48 sheep is $8\frac{1}{3}\%$ of a flock. How many sheep in the flock?

7. I spend \$55 a year for clothes; this is 10% of my salary. What is my salary?

8. I sold goods at a loss of $12\frac{1}{2}\%$, thereby losing \$375. What did the goods cost me?

140. To Find What Per Cent One Number is of Another.—We have been putting the "%" in the first member of the equation. In the following problems, we will want *per cent* in the answer; but the answer always is obtained in the second member of the equation. *Therefore, we will put the "%" in the second member.*

EXAMPLES.

1. \$7 is what % of \$20?

Solution: (1) \$20 = 100% of \$20. (Basis.)
$\frac{1}{20}$ of (1) = (2) \$1 = 5% of \$20.
7 × (2) = (3) \$7 = 35% of \$20, ans.

NOTE.—The number after the word "of" in the problem is the number used in the *Basis*.

2. A lawyer gets \$35 commission for collecting a debt of \$500. What % commission does he charge?

Solution: (1) \$500 = 100% of debt. (Basis.)
$\frac{1}{500}$ of (1) = (2) \$1 = $\frac{1}{5}$% of debt.
35 × (2) = (3) \$35 = 7% of debt, ans.

3. I paid \$52 for a buggy, and sold it so as to gain \$13. Find the % of profit.

NOTE.—As *profit and loss* is some per cent of the *cost*, in this problem we really want to know how many % \$13 is of the cost, or \$52.

Solution: (1) \$52 = 100% of cost. (Basis.)
$\frac{1}{4}$ of (1) = (2) \$13 = 25% of cost, ans.

4. I paid \$40 for a cow, and sold her so as to lose \$8. Find the % of loss.

Solution: (1) \$40 = 100% of cost. (Basis.)
$\frac{1}{5}$ of (1) = (2) \$8 = 20% of cost, ans.

EXERCISE CXXXI.

1. 10 is what % of 30?

2. \$24 is what % of \$40?

3. \$35 is what % of \$84?

4. I bought a wagon for \$60, and sold it for \$40. The selling price is what % of the cost price?

5. I bought goods for \$720, and sold them so as to gain \$144. Find my % of profit.

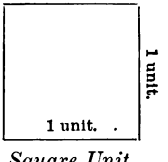
6. I bought goods for \$600, and sold them so as to lose \$100. Find my % of loss.

7. A lawyer charges \$50 for collecting \$600. Find his % of commission.

8. A man lost 60 sheep out of a flock of 360. Find his % of loss.

XII. MEASUREMENTS OF SURFACES AND SOLIDS.

141. The Units of Area and Volume.—A Surface is that which has only *length* and *width*. The number of square units in a surface is its **Area.** A **Square Unit** is the amount of a surface 1 unit long and 1 unit wide.

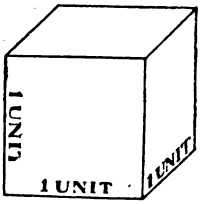


Square Unit.

If we select some linear unit (say) the *foot*, the *Unit Equation of Area* may be expressed thus:

Area of surface 1 ft. long, 1 ft. wide, = 1 square foot (sq. ft.).

A **Solid** is that which has *length*, *width*, and *thickness*. The number of cubic units in a solid is its **Volume.** A **Cubic Unit** is the amount of a solid 1 unit long, 1 unit wide, and 1 unit thick.



Cubic Unit.

If we select some linear unit (say) the *foot*, the *Unit Equation of Volume* may be expressed thus:

Volume of a Solid 1 ft. long, 1 ft. wide, 1 ft. thick=1 cubic foot (cu. ft.).

EXERCISE CXXXII.

Write the unit equation of area, using as the unit—

1. inch	*3.* yard	*5.* rod
2. foot	*4.* chain	*6.* mile

Write the unit equation of volume, using as the unit—

7. inch
8. foot
9. yard

142. Products and Parts of Equations of Area and Volume.—The *area*, depending upon both the *length* and *width*, is increased or diminished by increasing or diminishing either of these dimensions.

The *volume*, depending upon the *length*, *width*, and *thickness*, is increased or diminished by increasing or diminishing any of these dimensions.

EXAMPLES.

1. The area of a surface 1 yd. long, 1 yd. wide=1 sq. yd. Multiply by 5 (result in 2 forms).

Process: (1) Area of a surface 1 yd. l., 1 yd. w.=1 sq. yd.
5×(1)=(2) Area of a surface 5 yd. l., 1 yd. w.=5 sq. yd.; or,
5×(1)=(3) Area of a surface 1 yd. l., 5 yd. w.=5 sq. yd.

NOTE.—Be careful to observe (1) that *length* does not necessarily mean the longer dimension, but simply one of the dimensions, as in the third equation; (2) that multiplying either dimension without multiplying the other dimension multiplies the *area*.

2. Area of a surface 8 ft. long, 4 ft. wide=32 sq. ft. Find $\frac{1}{4}$ of it. (2 forms.)

Process: (1) Area of a surface 8 ft. l., 4 ft. w.=32 sq. ft.
$\frac{1}{4}$ of (1)=(2) Area of a surface 8 ft. l., 1 ft. w.=8 sq. ft.; or,
$\frac{1}{4}$ of (1)=(3) Area of a surface 2 ft. l., 4 ft. w.=8 sq. ft.

NOTE.—Taking $\frac{1}{4}$ of either dimension without changing the other dimension takes $\frac{1}{4}$ of the *area*.

3. Volume of a solid 1 ft. l., 1 ft. w., 1 ft. th.=1 cu. ft. Multiply by 7. (3 forms.)

Process: (1) Vol. of solid 1 ft. l., 1 ft. w., 1 ft. th.=1 cu. ft.
7×(1)=(2) Vol. of solid 7 ft. l., 1 ft. w., 1 ft. th.=7 cu. ft.; or,
7×(1)=(3) Vol. of solid 1 ft. l., 7 ft. w., 1 ft. th.=7 cu. ft.; or,
7×(1)=(4) Vol. of solid 1 ft. l., 1 ft. w., 7 ft. th.=7 cu. ft.

4. Volume of a solid 12 ft. l., 9 ft. w., 6 ft. th.=648 cu. ft. Find $\frac{1}{3}$ of it. (3 forms.)

Process: (1) Vol. of solid 12 ft. l., 9 ft. w., 6 ft. th.=648 cu. ft.
$\frac{1}{3}$ of (1)=(2) Vol. of solid 4 ft. l., 9 ft. w., 6 ft. th.=216 cu. ft.; or,
$\frac{1}{3}$ of (1)=(3) Vol. of solid 12 ft. l., 3 ft. w., 6 ft. th.=216 cu. ft.; or,
$\frac{1}{3}$ of (1)=(4) Vol. of solid 12 ft. l., 9 ft. w., 2 ft. th.=216 cu. ft.

EXERCISE CXXXIII.

1. Area of surface 5 ft. l., 3 ft. w. = 15 sq. ft. Multiply by 12. (2 forms.)

2. Area of surface 80 rd. l., 40 rd. w. = 20 acres. Find $\frac{1}{20}$ of it. (2 forms.)

3. Volume of solid 3 yd. l., 2 yd. w., 1 yd. th. = 6 cu. yd. Multiply by 8. (3 forms.)

4. Volume of solid 8 in. l., 6 in. w., 4 in. th. = 192 cu. in. Find $\frac{1}{2}$ of it. (3 forms.)

143. Find Areas and Volumes.—

EXAMPLES.

1. Find the area of a surface 12 ft. long and 8 ft. wide.

Solution: (1) Area of a surface 1 ft. l., 1 ft. w. = 1 sq. ft. (Basis.)
12 × (1) = (2) Area of a surface 12 ft. l., 1 ft. w. = 12 sq. ft.
8 × (2) = (3) Area of a surface 12 ft. l., 8 ft. w. = 96 sq. ft., ans.

2. How many cubic inches in a block of marble 30 in. long, 15 in. wide, and 9 in. thick?

Solution: (1) Vol. of a solid 1 in. l., 1 in. w., 1 in. th. = 1 cu. in. (Basis.)
30 × (1) = (2) Vol. of a solid 30 in. l., 1 in. w., 1 in. th. = 30 cu. in.
15 × (2) = (3) Vol. of a solid 30 in. l., 15 in. w., 1 in. th. = 450 cu. in.
9 × (3) = (4) Vol. of a solid 30 in. l., 15 in. w., 9 in. th. = 4050 cu. in., ans.

EXERCISE CXXXIV.

1. How many square rods in a lawn 20 rods long and 16 rods wide?

2. How many cubic inches in a brick 9 inches long, 4 inches wide, and 2 inches thick?

3. What is the area of a floor 30 ft. long and 18 ft. wide?

4. How many square yards of plastering on a wall 10 yards long and 3 yards high?

5. How many cubic feet in a block of marble 6 ft. long, 3 ft. wide, and 1½ ft. thick?

6. How many square yards in a ceiling 18 feet long and 15 feet wide?

NOTE.—Reduce the feet to yards before attempting to find the area.

144. Tables of Surface Measures.—There are two tables of surface measures, corresponding to the two tables of linear measures. The one is the **Common Square Measures,** used to measure the area of such surfaces as floors, ceilings, carpets, cloth, and the like.

TABLE.

1 sq. yd.=9 sq. ft.
1 sq. ft.=144 sq. in.

The other table is the **Surveyors' Square Measures,** used in measuring the area of land.

TABLE.

1 township (Tp.)=36 sq. mi., or Sections.
1 sq. mi.=640 acres (A.).
1 A.=160 sq. rd.
1 A.=10 sq. ch.
1 sq. ch.=16 sq. rd.

EXERCISE CXXXV.

1. Reduce 12 sq. yd. 3 sq. ft. to square inches.
2. Reduce 18 sq. yd. to square inches.
3. How many sq. rd. in $5\frac{1}{2}$ acres?
4. What are 2 Tp. of land worth, at $25 per acre?
5. Reduce 1280 sq. rd. to acres.
6. Reduce 17344 sq. chains to higher units.
7. Reduce 3420 sq. in. to higher units.
8. What will it cost to plaster a wall 10 ft. high and 18 ft. long, at 25¢ per sq. yd.?
9. A floor is 12 ft. wide and 18 ft. long. What will a carpet for it cost, at $.55 per sq. yd.?
10. What will a walk 300 ft. long, 4 ft. wide, cost, at 20¢ per sq. foot?

145. Table of Solid Measures.—This table is used in measuring volumes of solids and the capacities of bins, tanks, and the like.

TABLE.

1 cu. yd. = 27 cu. ft.
1 cu. ft. = 1728 cu. in.

NOTE.—A gallon contains 231 cu. in.; a bushel contains 2150.4 cu. in.

EXAMPLE.

What is the difference in cu. in. between a dry-measure quart and a liquid-measure quart?

Dry Quart.

Solution: (1) 1 qt. = $\frac{1}{8}$ pk. (Basis.)
(2) 1 pk. = $\frac{1}{4}$ bu. (Basis.)
$\frac{1}{8}$ of (2) = (3) $\frac{1}{8}$ pk. = $\frac{1}{32}$ bu.
(4) 1 bu. = 2150.4 cu. in. (Basis.)
$\frac{1}{32}$ of (4) = (5) $\frac{1}{32}$ bu. = 67.2 cu. in.

Liquid Quart.

(6) 1 qt. = $\frac{1}{4}$ gal. (Basis.)
(7) 1 gal. = 231 cu. in. (Basis.)
$\frac{1}{4}$ of (7) = (8) $\frac{1}{4}$ gal. = 57.75 cu. in.

(9) 67.2 cu. in. − 57.75 cu. in. = 9.45 cu. in., answer, in favor of the dry-measure quart.

EXERCISE CXXXVI.

1. Reduce 4 cu. yd. to cu. in.
2. Reduce 7 cu. yd. 17 cu. ft. 375 cu. in. to cu. in.
3. Reduce 175 cu. in. to the fraction of a cu. ft.
4. Reduce 300 cu. in. to the fraction of a cu. yd.
5. Reduce 16753 cu. in. to higher units.
6. A bin is 10 ft. long, 18 ft. wide, and 6 ft. high (inside measures). How many bushels will it hold? (Answer true to 1 decimal place.)

NOTE.—(1) Get the area in cu. ft.; (2) reduce the cu. ft. to cu. in.; and (3) reduce the cu. in. to bushels.

7. A revolving churn is 4 ft. long, 3 ft. wide, and 3 ft. deep (inside measurements). How many gallons of milk will it hold?

8. What will be the cost of digging a cellar 20 ft. long, 15 ft. wide, and 6 ft. deep, at $1.25 per cubic yard?

XIII. SIMPLE INTEREST, AND THE WORK PROBLEMS.

146. Terms Used in Interest.—Interest is a sum paid for the use of money. The amount of money used is the **Principal.** The per cent of the principal which equals one year's interest is the **Rate.** If only the principal bears interest, the interest is **Simple Interest.**

147. Time.—Since the rate in interest problems is "*per annum*" (by the year), the *time* should be expressed as years. *Months* are 12ths of a year, and *days* are 360ths (365ths in exact interest) of a year. A little drill, such as is given in the following examples, will give speed and ease in expressing *time* as *years*.

EXAMPLES.

1. Express as fractions of a year: (1) 1 mo., (2) 3 mo., (3) 5 mo., (4) 7 mo., (5) 9 mo., (6) 11 mo.

(1) 1 mo. = $\frac{1}{12}$ yr.
(2) 3 mo. = $\frac{3}{12}$ yr. = $\frac{1}{4}$ yr.
(3) 5 mo. = $\frac{5}{12}$ yr.
(4) 7 mo. = $\frac{7}{12}$ year.
(5) 9 mo. = $\frac{9}{12}$ yr. = $\frac{3}{4}$ yr.
(6) 11 mo. = $\frac{11}{12}$ yr.

2. Express as fractions of a year: (1) 3 da., (2) 10 da., (3) 15 da., (4) 18 da., (5) 33 da., (6) 93 da.

(1) 3 da. = $\frac{3}{360}$ yr. = $\frac{1}{120}$ yr.
(2) 10 da. = $\frac{10}{360}$ yr. = $\frac{1}{36}$ yr.
(3) 15 da. = $\frac{15}{360}$ yr. = $\frac{1}{24}$ yr.
(4) 18 da. = $\frac{18}{360}$ yr. = $\frac{1}{20}$ yr.
(5) 33 da. = $\frac{33}{360}$ yr. = $\frac{11}{120}$ yr.
(6) 93 da. = $\frac{93}{360}$ yr. = $\frac{31}{120}$ yr.

3. Express as fractions of a year: (1) 2 mo. 3 da.; (2) 3 mo. 15 da.; (3) 5 mo. 18 da.; (4) 6 mo. 8 da.

(1) 2 mo. 3 da. $= \frac{63}{360}$ yr. $= \frac{7}{40}$ yr. (3) 5 mo. 18 da. $= \frac{198}{360}$ yr. $= \frac{7}{18}$ yr.
(2) 3 mo. 15 da. $= \frac{105}{360}$ yr. $= \frac{7}{24}$ yr. (4) 6 mo. 8 da. $= \frac{188}{360}$ yr. $= \frac{47}{90}$ yr.

NOTE.—Do work mentally, and write only results.

148. The Unit Equation of Interest.—Since in interest problems, a certain *principal* bears *interest* at a certain *rate* for the *time* it is used, there are four elements or numbers involved in the *interest equation*—*principal*, *time*, *rate*, and *interest*. Thus:

Interest on \$1 for 1 year at 1% = \$.01.

Multiplying the principal, time, or rate, multiplies the interest; taking a part of the principal, time or rate, takes such a part of the interest.

EXAMPLES.

1. Find the interest on \$300 at 10% for 2 years.

Solution: (1) Int. on \$1 for 1 yr. at 1% = \$.01. (Basis.)
300 × (1) = (2) Int. on \$300 for 1 yr. at 1% = \$3.
10 × (2) = (3) Int. on \$300 for 1 yr. at 10% = \$30.
2 × (3) = (4) Int. on \$300 for 2 yr. at 10% = \$60, ans.

2. Find the interest on \$800 for 1 yr. 6 mo. at 10%.

Solution: (1) 1 yr. 6 mo. $= 1\frac{1}{2}$ yr.
(2) Int. on \$1 for 1 yr. at 1% = \$.01. (Basis.)
800 × (2) = (3) Int. on \$800 for 1 yr. at 1% = \$8.
$1\frac{1}{2}$ × (3) = (4) Int. on \$800 for $1\frac{1}{2}$ yr. at 1% = \$12.
10 × (4) = (5) Int. on \$800 for $1\frac{1}{2}$ yr. at 10% = \$120, ans.

3. Find the interest on \$500 for 90 days at 6%.

Solution: (1) 90 days = $\frac{1}{4}$ yr.
(2) Int. on \$1 at 1% for 1 yr. = \$.01. (Basis.)
500 × (2) = (3) Int. on \$500 at 1% for 1 yr. = \$5.
6 × (3) = (4) Int. on \$500 at 6% for 1 yr. = \$30.
$\frac{1}{4}$ of (4) = (5) Int. on \$500 at 6% for $\frac{1}{4}$ yr. = \$7.50, ans.

4. Find the interest on \$400 at 8% for 2 yr. 4 mo. 24 days.

Solution : (1) 2 yr. 4 mo. 24 da. = $2\frac{2}{3}$ yr.
(2) Int. on \$1 for 1 yr. at 1% = \$.01. (Basis.)
400 × (2) = (3) Int. on \$400 for 1 yr. at 1% = \$4.
$2\frac{2}{3}$ × (3) = (4) Int. on \$300 for $2\frac{2}{3}$ yr. at 1% = \$9.60.
8 × (4) = (5) Int. on \$400 for $2\frac{2}{3}$ yr. at 8% = \$76.80, ans.

EXERCISE CXXXVII.

1. Find the interest on \$460 for 3 years at 6%.
2. Find the interest on \$840 for 5 years at 7%.
3. Find the interest on \$1260 for 4 years at 5%.
4. Find the interest on \$700 for 2 yr. 3 mo. at 10%.
5. Find the interest on \$600 for 1 yr. 6 mo. at 6%.
6. Find the interest on \$900 for 8 mo. at 9%.
7. Find the interest on \$720 for 1 yr. 4 mo. at 6%.
8. Find the interest on \$360 for 93 days at 6%.
9. Find the interest on \$480 for 4 mo. 12 da. at 10%.
10. Find the interest on \$1000 for 1 yr. 2 mo. 18 da. at 6%.

[A NOTE.]

DENVER, COLO., Jan. 17, 1900.

Ninety days after date I promise to pay C. D. Shukers, ——— or order, at THE FIRST NATIONAL BANK OF DENVER, Three Hundred ——— no/100 DOLLARS, with interest at the rate of ten per cent per annum until paid, for value paid.

\$300.00 W. S. Fitzpatrick.

NOTE.—This note is legally due 93 days after Jan. 17, or April 20, and will bear interest for that time. In Texas and in many other States, the holder of a note must allow 3 days more than the time stated in the note for payment. These are called *days of grace.*

11. How much will Mr. Fitzpatrick owe Mr. Shukers on above note when it becomes due?

12. Write a note, in which you promise to pay Mr. Fitzpatrick $540 in 4 months, with interest at 8%. Make your note payable at your nearest bank.

NOTE.—Teacher should explain the meaning of every part of a promissory note to the pupil.

149. Work Problems.—These problems are somewhat similar in nature to the interest problems.

PROBLEM.

If 5 men cut 15 cords of wood in 2 days, how many cords can 8 men cut in 11 days?

Solution: (1) Amount cut by 5 men in 2 da. = 15 cords. (Basis.)
$\frac{1}{5}$ of (1) = (2) Amount cut by 1 man in 2 da. = 3 cords.
$\frac{1}{2}$ of (2) = (3) Amount cut by 1 man in 1 da. = $1\frac{1}{2}$ cords.
8 × (3) = (4) Amount cut by 8 men in 1 da. = 12 cords.
11 × (4) = (5) Amount cut by 8 men in 11 da. = 132 cords, ans.

EXERCISE CXXXVIII.

1. Length of a wall built by 6 men in 8 days is 528 ft.; how long a wall can 5 men build in 13 days?

2. If 3 men build 36 rods of fence in 6 days, how many rods can 8 men build in 7 days?

3. If 3 blacksmiths shoe 18 horses in 1 day, how many horses can 8 men shoe in 5 days?

4. If 1 sewing machine takes 240 stitches in 2 minutes, at that rate how many stitches will 5 machines take in 1 hour?

5. If 16 men build 72 rods of fence in 8 days, how many rods can 96 men build in 8 days?

6. If $200 gain $30 interest in 1 yr. 6 mo., what amount of interest will $3400 gain in 2 yr. 9 mo.?

NOTE.—Reduce years and months to months.

7. If 6 men spend $300 in 8 mo., how many dollars will 15 men spend in 20 months?

8. If 3 boys eat 18 apples in 1 day, how many apples will 5 boys eat in 10 days?

9. If 8 men build 400 feet of wall in 10 days, how many feet of wall can 18 men build in 9 days?

10. If 16 men hoe 60 acres of corn in 20 days, how many acres can 36 men hoe in 12 days?

XIV. PROBLEMS CLASSIFIED.

Explanation.—The problems of elementary arithmetic, though various in their forms of expression, may be classified into a few distinct types or classes. It is certainly an appropriate conclusion to such an arithmetic to present this classification for study.

It will greatly assist the pupil in fixing these types in his memory, if, in addition to the exercises required in the text, the teacher will require him to classify many problems from different parts of this and other texts.

Questions, which by their terms require only fundamental or derived operations to be performed, and which are questions of *process* only, are not considered in this classification.

150. Class I.—*Given a number, to find* (1) *a part of it, or* (2) *a multiple of it.* (Commit.)

To find a part: EXAMPLES.

1. James has $720, and John has $\frac{3}{5}$ as much. How much has John?

2. Find 20% of $600.

3. What cost $\frac{3}{4}$ ton of coal, at $3.80 per ton?

To find a multiple:

1. Clara is 12 years old, and her mother is 4 times as old. How old is the mother?

2. Find 300% of $240.

3. What will 10 hats cost, at $2.40 each?

EXERCISE CXXXIX.

1. Solve the examples given above.
2. Make 3 examples, requiring a part to be found.
3. Make 3 examples, requiring a multiple to be found.

151. Class II.—*Given a part of a number, to find* (1) *the number,* (2) *a multiple of the number, or* (3) *another part of the number.* (Commit.)

EXAMPLES.

To find the number:

1. $\frac{5}{8}$ of my salary is \$375. What is my salary?
2. 15% of a certain debt is \$630. Find the debt.
3. If $\frac{11}{17}$ of a boat is worth \$9900, what is the whole boat worth?

To find a multiple:

4. In $\frac{3}{11}$ of a load of corn there are 6 bu. How many bushels in 25 such loads?
5. $8\frac{1}{3}$% of a certain sum is \$50. What would 275% of that sum be?
6. If $\frac{2}{3}$ of a bolt of cloth is worth \$3.20, what are 13 such bolts worth?

To find another part:

7. If $\frac{7}{8}$ of Mr. Brown's capital is \$700, how much is $\frac{1}{8}$ of it?
8. I deposit 48% of my money, which is \$960, in the bank, and invest 28% of my money in calves. How much do I spend for calves?
9. $\frac{5}{8}$ of an article is worth \$.75; what is $\frac{1}{12}$ of it worth?

EXERCISE CXL.

1. Solve the examples given above.
2. Make 3 examples, requiring the number to be found.
3. Make 3 examples, requiring a multiple to be found.
4. Make 3 examples, requiring another part to be found.

152. Class III.—*Given a multiple of a number, to find* (1) *the number*, (2) *a part of the number*, *or* (3) *another multiple.*

To find the number: EXAMPLES.

1. If 6 men plow 45 acres in 3 days, how many acres will 1 man plow in 1 day?

2. 360% of a number is 252. Find the number.

3. If 20 bu. of apples cost $18, what is the price per bushel?

To find a part:

4. If 25 boxes of pens are worth $7.50, what is $\frac{1}{3}$ of a box worth?

5. If 5 men can saw 20 cords of wood in 4 days, how many cords can 1 man saw in $\frac{1}{2}$ day?

6. 120% of a number is 72. Find 35% of it.

To find another multiple:

7. If 11 men set 660 pages of type in 6 days, how many pages will 21 men set in 8 days?

8. 240% of my age is 84 yr. Find 380% of it.

9. If 75 bu. of wheat are worth $60, what are 42 bu. worth?

EXERCISE CLXI.

1. Solve the examples given above.

2. Make 3 examples, requiring the number to be found.

3. Make 3 examples, requiring a part to be found.

4. Make 3 examples, requiring another multiple to be found.

153. Class IV.—*Given two numbers, to find* (1) *what part the one is of the other, or* (2) *what multiple the one is of the other.* (Commit.)

To find what part: EXAMPLES.

1. 21 is what part of 56?

2. A lawyer charges $35 for collecting a debt of $700. What % does he charge?

3. I bought an article for $5 and sold it so as to lose $3. What part of my investment did I lose?

To find what multiple:

4. 760 is how many times 38?

5. I bought an article for $5 and sold it for $7.50. The selling price is what % of the cost price?

6. Charles is 12 years old and George is 30 years old. George is how many times as old as Charles?

EXERCISE CXLII.

1. Solve the examples given above.

2. Make 3 examples, requiring *what part* to be found.

3. Make 3 examples, requiring *what multiple* to be found.